

# IMPLEMENTATION AND APPLICATION OF MODULAR THREE DIMENSIONAL FINITE DIFFERENCE GROUNDWATER FLOW MODEL

*A Thesis Submitted  
In Partial fulfilment of the Requirements  
for the Degree of*

**MASTER OF TECHNOLOGY**

*by*

**Y. SIVARAMAIAH**

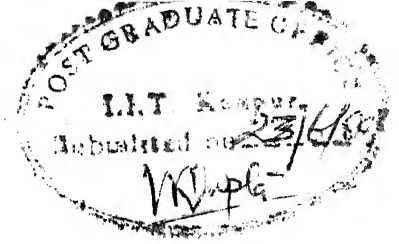
*to the*

**DEPARTMENT OF CIVIL ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY, KANPUR**

**JUNE, 1989**

to  
my beloved  
parents  
and  
guide



**CERTIFICATE**

This is to certify that the thesis "Implementation and Application of Modular Three Dimensional Finite Difference Groundwater Flow Model" submitted by Shri Y.Sivaramaiah, in partial fulfilment of Master of Technology of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under my supervision and guidance. This work has not been submitted elsewhere for a degree.

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## ABSTRACT

There are a number of studies in the literature where two dimensional finite difference analysis of complex groundwater basins are reported. However, not many studies on three dimensional models are available. In the present study a modular three dimensional finite difference model developed by the US Geological Survey is implemented on a mini computer namely Micro VAX II system and also on DEC 1090. The model is then applied to a small basin in Anantapur district of Andhra Pradesh. The model being a sophisticated one, requires substantial amount of data for a few years. However, the data for only two years were available. Hence, the results are likely to be approximate. The main objective of the study was to demonstrate the applicability of three dimensional model to a groundwater basin.

In addition to three dimensional finite difference model, the hydrological budget method is also used for the basin, as the basin is small. The advantage of the finite difference model over the hydrologic budget method is that it can take into account the spatial variation of hydrological and geological parameters and also complex boundary conditions.

The results of the three dimensional finite difference model are voluminous. Only sample values of heads for layer number three at the end of December 1979 are reported in the thesis.

## CHAPTER I

### INTRODUCTION

#### 1.1 General:

For the regions where the rainfall is very less, the availability of surface water becomes scarce. For such regions it is always advisable to study the possibility of groundwater resources and a proper development of the same should be considered.

In planning and development of groundwater, care should be excercised to avoid undesirable effects of over-development. Some of the undesirable effects are:

1. Gradual lowering of piezometric surface or groundwater table.
2. Encroachment of saline water.
3. Subsidence of land, etc.

If groundwater is developed indiscriminately, it is possible that the amount of water withdrawn would be larger than the long term average groundwater recharged to the groundwater basin. This will result in a gradual decline of piezometric surface or groundwater table causing many shallow wells to go dry and making groundwater lift larger in deeper wells resulting in increased pumping cost. If the well fields are near sea coast, over development will cause encroachment of sea water into fresh water. Further, there is evidence of land subsidence in reaches where the aquifers are intersperced with compressible silty and clayey layers, from which large scale goundwater development has taken place.

Thus, whenever groundwater development is considered, proper planning is required to avoid undesirable effects mentioned above. Such planning requires making suitable mathematical models of the basin and analysing the effects of development of groundwater. A large number of two dimensional mathematical models have been developed and applied to the groundwater basins. Relatively fewer number of three dimensional modeling methods have been reported in the literature. Recently, a general modular three dimensional finite difference groundwater flow model has been developed by the US Geological Survey (1984). This modular program is quite versatile. In the present study this program was implemented on the Micro Vax II system. After fully implementing the program it was applied to one of the basins in the Anantapur district of Andhra Pradesh for which some data was available.

## 1.2 Objectives of the study:

Given the data like rainfall, evaporation, canal flows, tank gauging observations, geological features and aquifer parameters, a model of groundwater basin can be constructed for evaluation of the groundwater potential and it can be simulated for various policy options and the spatial distribution of the heads can be predicted.

The basin can be studied by simple hydrologic budgeting method for the basin as a single unit or by a mathematical model. The first method is very simple and hydrological balance was evaluated by considering simple analysis, where as in the detailed mathematical analysis a model of groundwater basin can be constructed regardless of its size and complexity. Even small details of geology and hydrology can be incorporated and studied by mathematical formulation. In constructing a groundwater basin



model, the basin is divided into smaller areas of rectangular or square blocks. The centre of the block is considered as a node for the mathematical treatment, assuming that the recharge or discharge into the model area occurs at that node. It is also assumed that the head in the block is same as that at the corresponding node. This gives the spatial variation (or distribution) of the groundwater.

The main objectives of the study are:

1. Implementation of the US Geological Survey modular three dimensional finite difference program.
2. Application of hydrologic budget method for the basin.
3. Application of the three dimensional modular program to the study area.

### 1.3 Scope of the study:

The US Geological Survey model is a very general model with several options. It uses the programming language Fortran 66. It is applicable without much modifications on main frame computers such as CDC, DEC, CRAY etc, using Fortran 77 compilers. In the present study the above program was implemented on a mini computer namely Micro Vax II. Modifications necessary for this were incorporated in this program.

Obviously for a three dimensional model the data requirement is much more comprehensive than for a two dimensional model. For example, we require not only storativity and transmissivity values at different points in lateral space but also at different points in the vertical direction. Such detailed data are usually not available for any basin. In this sense the three dimensional model can only be applied to basins which are taken up for research study. It is however possible to use this program for two

dimensional problems also by using suitable options provided in the program. In the present study, therefore, the first objective was to implement the three dimensional program on the Micro Vax II and DEC 1090 the second objective was to apply this to a small catchment in Anantapur district of Andhra Pradesh. Further, the simple hydrological budget method is also applied for the basin.

All the data that was available for this catchment was obtained. Based on the Lithology of the area, the data was collected from four different depths (layers). The length of the data was very short, only two years (1979,1980). Hence the results of application of the model will be approximate. The aim of the study is to demonstrate the applicability of the three dimensional model rather than to get reliable results for the basin which would not be possible because of the paucity of data.

#### 1.4 Organization of the study:

This thesis has been divided into five chapters:

- i. Chapter 1 gives a general introduction about the objectives and scope of the study.
- ii. Necessary review of literature is presented in Chapter 2.
- iii. Chapter 3 deals with the implementation of the program.
- iv. Application of a simple hydrologic model and the three dimensional finite difference groundwater flow model are discussed in Chapter 4.
- v. Results, conclusions and suggestions for further study are presented in Chapter 5.

## CHAPTER 2

### REVIEW OF LITERATURE

A model is a tool designed to represent a simplified version of reality (Wang and Anderson, 1982). A mathematical model consists of a set of differential equations that are known to govern the flow of groundwater. The reliability of predictions from a groundwater model depends on how well the model approximates the field situation.

Since no model can really reproduce all the variety in a real groundwater basin it is of some value to recognise how good the model may be for the purpose intended. The model itself can be used to reach a better understanding of the various aspects and phenomena involved in the question of model suitability (Thomas, 1973).

Several types of models can be used to study groundwater flow systems. They can be divided into three broad categories (Prickett, 1975): sand tank models, analog models (including viscous fluid models, and electrical models) and mathematical models (including analytical and numerical models). A sand tank model consists of a tank filled with an unconsolidated porous medium through which water is allowed to flow. A major drawback of sand tank model is the problem of scaling down a field situation to the dimension of a laboratory model. The observations measured at the scale of a tank model are often different from conditions observed in the field. Viscous fluid models known as the H<sub>2</sub>O-H<sub>2</sub>O or parallel plate models in which viscous fluid is made to flow

between two closely spaced parallel plates and the observations are made. Electrical analog model consists of boards wired with electrical networks of resistors and capacitors. They work according to the principle that the flow of groundwater is analogous to the flow of electricity. This analogy is expressed in mathematical similarity between Darcy's law for groundwater flow and Ohm's law for the flow of electricity. Changes in voltage in electrical models are analogous to changes in groundwater head. A drawback in electric analog model is that it is unique for each aquifer system. When a different aquifer is to be used, an entirely new electrical analog model must be built. A mathematical model consists of differential equations that are known to govern the flow of groundwater.

Though sand tank models and analog models were extensively used prior to 1960 and ever since then, due to the wide availability of high speed digital computers, mathematical models have been favoured. The solution to the mathematical models can be obtained either by analytical or by numerical techniques. Unfortunately the equations of flow and continuity in the form of partial differential equations do not lend themselves easily to rigorous analytical solutions where boundaries are complicated. Further, the user must have a sufficient mathematical background to compute such solutions (Wang and Anderson, 1982). For example, Toth (1962) presented an analytical solution to the simple regional groundwater flow. The model consisted of a topographic high on one side and a valley bottom on the other, where water was discharging and the water table was gently sloping towards the valley. To obtain this analytical solution Toth (1962) introduced

several simplifying assumptions like the medium is an homogeneous, isotropic aquifer with linear water table configuration and he approximated the problem domain by a rectangle. None of these assumptions are necessary to obtain a numerical solution. To obtain solutions for such a simple problem the analytical method is complicated and for very complex groundwater problem it is not always possible to obtain a solution. To overcome this difficulty, one should resort to approximate methods using numerical techniques.

At present there are two well known numerical methods which are in wide usage. They are the Finite Element Method (FEM) and the Finite Difference Method (FDM). One more method namely the Boundary Integral Equation Method (BIEM) is also being used. Of all the numerical methods, the finite difference method is the oldest and the most widely used technique. Ever since the classical paper by Richardson (1910), introducing finite difference as a method to calculate approximately the solution of partial differential equations was presented and later Shaw and Southwell (1941) applied this method for the solution of steady state seepage problem, this method has been extensively used in groundwater flow.

The partial differential equation governing the flow of groundwater can be approximated numerically by finite difference technique or finite element method. In doing so, one replaces continuous variables with discrete variables that are defined at grid blocks (or nodes). Thus the continuous differential equation defining hydraulic head everywhere in the aquifer is replaced by a finite number of algebraic equations that define hydraulic head at

specific points. The system of algebraic equations is generally solved using matrix techniques. This approach constitutes a numerical model and generally a computer program is written to solve the equations on a digital computer.

Following Shah and Southwell (1941), Staltman (1956) and Fayers and Sheldon (1962) were among the early users of finite difference-based numerical method for studying the regional groundwater basins. Since then, there have been many developments in numerical methods based on finite difference approach. Tyson and Weber (1964) used non-uniform polygonal discretization of the aquifer area and the resulting finite-difference equations were solved by Gauss-Seidel method. In this study, a groundwater model was developed for a non-homogeneous but isotropic aquifer under confined or unconfined condition. The model was used for predicting the dynamic behaviour of a regional groundwater basin.

Based on the changes of homogeneity at different places of a basin, Fiering (1964) used a variable size rectangular grid while discretizing a nonhomogeneous and isotropic aquifer in both confined and unconfined situation. The set of finite difference equations was solved by an iterative technique. The model was aimed at predicting drawdown in the aquifer under variable well pumping rates and recharges. By a similar kind of model in which Gauss-Seidel iterative technique was used to solve the set of difference equations, Remson et.al. (1966) analysed the recharge effects of a proposed reservoir on a groundwater basin.

Peaceman and Rachford (1955) proposed a new method which has been found to result in significant advantages in solving groundwater flow problems. This was named as Alternate Direction Implicit (ADI) Method and in the later years the iterative ADI methods became very popular to solve the set of simultaneous finite-difference equations (Pinder and Bredehoeft, 1968; Prickett and Lonnquist, 1968; Pinder, 1970; Bredehoeft and Pinder, 1970; Pinder and Cooper, 1970; Rushton and Tomlinson, 1971; Bredehoeft and Pinder, 1973; Lin, 1973). In these studies various types of groundwater problems such as transient flow in multiple layered aquifer involving storage in confining layers, effect of dispersion on steady movement of salt water front, and predicting future movements of contaminants were studied for various types of aquifer situations.

A three dimensional model was described by Freeze (1971) which could handle heterogeneous, anisotropic, multilayer saturated-unsaturated and confined-unconfined groundwater basins. The nodal sizes do not have to be constant but they must be block shaped (rectangular in two dimensions). This paper describes the use of the line successive over-relaxation technique to solve the problem numerically. Another interesting finite difference technique was discussed by Bibby and Sunada (1971) where the implicit central finite difference scheme was used and a solution was obtained by the Gaussian elimination procedure. This particular model was prepared to handle the leaky aquifer situation.

Any traditional approach may be appropriate for a system governed by a single partial differential equation but for a system comprising portions governed by effectively different equations may make the modeling process difficult. For any real groundwater system, it is likely that detailed modeling may require extensive computer capacity, CPU time and considerable amount of input data which consequently may prove to be an important restriction (Maddock, 1973). In particular, this difficulty stands for a large scale aquifer system where direct use of traditional technique may prove to be inadequate. The above difficulty is overcome by the method of decomposition (Dreizen and Haines, 1977; Sharma and Lakshminarayana, 1986). In their study a large and complex aquifer system was decomposed into a number of subareas according to certain considerations involving geographical, geological and hydrological characteristics, administrative and operational judgements, or any other requirements associated with the particular need for the groundwater simulation model.

Estimation of groundwater recharge is one of the important components of groundwater studies. Problems were encountered by Rushton et al. (1988), when trying to construct a recharge/abstraction balance for the Liverpool sandstone aquifer. Potential recharge rates were calculated by a soil moisture balance method (Rushton and Ward, 1979), yet actual recharge to the aquifer had to be reduced by a factor which was dependent on the thickness and clay content of drift deposits lying between the soil and the aquifer. This was complicated further at grid points where the water table lay within the drift profile. This highlights the need for comprehensive geological survey as well as water flux measurement to get a true picture of recharge to an aquifer.



The use of surface infiltration as a measure of recharge leads into with difficulties and uncertainties, partly because of the artificial effects of ponding, which is necessary for volumetric work and for ensuring potential rates of infiltration. Also, surface infiltration is generally more variable than transmissivity lower down the profile, which results from the nature of the surface, controlled as it is by the cumulative effect of raindrop and human compaction of the natural soil surface structure, and by the vegetation cover.

A better approach involves estimation of soil water fluxes, allowing recharge to be estimated by the water balance equation. It is generally recognized that climate is one of the most important factors determining the amount of water loss by evapotranspiration from the crop. Apart from the climatic factors, evapotranspiration for a given crop is also determined by the type of crop and so are growth characteristics. Local environment, soil and soil water conditions, fertilizers, insect and disease infestations, agricultural and irrigation practices and other factors may also influence growth rates and result in evapotranspiration (FAO Irrigation and Drainage Paper 24, 1975).

Methods are used to predict evapotranspiration from climatic variables owing to the difficulty of obtaining accurate direct measurements under field conditions. Most prediction formulae use a differentiation between the components of climate and crop. Such formulae often have to be applied under climatic and agronomic conditions very different to those for which they were originally developed.

Evaporation pans provide a measurement of the integrated effect of radiation, wind, temperature and humidity on evaporation from an open water surface. In a similar fashion the plant responds to the same climatic variables but several major factors may produce significant differences in loss of water. Reflectivity of radiation from a water surface is only 5-8%, while that from most vegetative surfaces is 20-25% of solar radiation received. Daytime storage of heat within the pan can be appreciable and may cause almost equal distribution of evaporation between night and day, while most crops lose 95% or more of their 24-hour loss during daytime hours.

Reference crop evapotranspiration ( $E_{To}$ ) can be predicted by

$$E_{To} = K_p * E_{pan}$$

where  $E_{To}$  is in mm/day,  $E_{pan}$  is pan evaporation in mm/day and represents the mean daily value of the period considered;  $K_p$  is the pan coefficient.

The values of  $K_p$  are given in the FAO Irrigation and Drainage Paper 24 (1975) for different conditions of humidity and wind, pan environment and type of pan and should be applied to pans located in an open environment with no crops taller than 1 m within some 50 m of the pan. Coefficients relating open water evaporation  $E_o$  to reference crop evapotranspiration  $E_{To}$  are presented in FAO Irrigation and Drainage Paper 24 (1975). These values apply to shallow reservoirs and lakes with depths of less than 5 m and can be used to compute monthly  $E_o$ , once monthly  $E_{To}$  has been determined. The presented values apply equally to deep reservoirs and lakes in equatorial zones.

Literature review on some case studies are reported in the following section.

Because of the importance of the integrated multi-disciplinary and basin wise groundwater potential, the CGWB (1983) has undertaken special projects based on socio-economic setup, the diverse hydrometeorological and hydrological settings for carrying out groundwater balance studies. The Noyil, Amaravati-Ponnani river basin projects of South India, for which extensive investigation was reported. The main source of recharge depends upon North-East monsoon where wide variation of monthly and annual rainfall and interannual variations are very high (ranging from 20% to 90%). The hydrological investigation preceded by well inventories (to a depth of 304m) indicated water bearing fractured zones. This type of report gives clear understanding of various aspects of groundwater balance evaluation and possible ways of improving the groundwater reserves. In this project the solution was arrived by using numerical techniques i.e. the finite-difference method.

Beken and Byloos (1977) applied a monthly water balance model to Grote Nete test basin (553 sq km) in the North of Belgium. This low region has a complex geological structure, its boundaries are more or less unknown and deep infiltration into a deep aquifer is most likely to occur. This area is covered by several navigation canals. The inputs are monthly precipitation and Penman potential evapotranspiration values. The model computes actual evapotranspiration, water storage in the basins, direct runoff and infiltration, base flow and total stream discharge, deep infiltration loss into the underlying aquifer and constant seepage from the canals. The author claims an accuracy of 0.2%.

Schicht and Walton (1961) conducted studies for three basins in Illinois. Groundwater recharge runoff and evapotranspiration were determined for the study basins, from hydrologic and groundwater budgets using precipitation, stream flow and groundwater level data. The study basins are in North-Central, West-Southwestren and East-Central Illinois. Groundwater runoff under flood hydrographs, during rainless periods, groundwater evapotranspiration were determined from rating curves of mean groundwater stage versus groundwater runoff. Groundwater storage changes were computed from data on changes in mean groundwater stage by using a graph showing the relationship between gravity yields and average time of drainage. The gravity yield-average time of drainage graphs were prepared from the results of hydrologic budget studies made for periods during winter and early spring months when evapotranspiration and soil moisture changes were very small. Actual evapotranspiration is appraised from annual hydrologic budgets and were compared to mean annual potential evapotranspiration and annual water laws.

The Central Ground Water Board (Southern Region, 1988) has drilled exploratory bore holes at different places in Ananthapur district of Andhra Pradesh and presented the lithology of the area. They have also conducted air tests and pumping tests to determine the aquifer parameters, also the drilling time logs of exploratory bore holes were reported as Basic Data Reports. These reports give some useful data necessary for the present study.

## CHAPTER 3

### IMPLEMENTATION OF MODULAR THREE DIMENSIONAL GROUNDWATER FLOW MODEL OF THE U.S. GEOLOGICAL SURVEY

#### 3.1 MATHEMATICAL MODEL:

The movement of groundwater of constant density through porous earth material may be described by the partial-differential equation

$$\frac{\partial}{\partial x} \left( K_{xx} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{yy} \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_{zz} \frac{\partial h}{\partial z} \right) - W = S_s \frac{\partial h}{\partial t} \quad (3.1)$$

Where

x, y and z are cartesian coordinates aligned along the major axes of hydraulic conductivity  $K_{xx}$ ,  $K_{yy}$ ,  $K_{zz}$ ;

h is the piezometric head (L);

W is a volumetric flux per unit volume and represents sources and /or sinks of water ( $T^{-1}$ );

$S_s$  is the specific storage of the porous material ( $L^{-1}$ ); and  
t is time (T).

In general,  $S_s$ ,  $K_{xx}$ ,  $K_{yy}$ ,  $K_{zz}$  may be functions of space ( $S_s = S_s(x, y, z)$ , and  $K_{xx} = K_{xx}(x, y, z)$ , etc.) and h and W may be functions of space and time ( $h = h(x, y, z, t)$ ,  $W = W(x, y, z, t)$ ) so that equation 3.1 describes groundwater flow under nonequilibrium conditions in a heterogenous and anisotropic medium.

In a finite difference model the domain is divided into a number of rectangular or square cells. Equation 3.1 is applied to each cell either by using finite differences for each differential term or by using a physical approach as below.

$$\Sigma q_1 = S_s \frac{\Delta h}{\Delta t} \Delta V \quad (3.2)$$

where  $q_1$  is a flow rate into the cell ( $L^3 t^{-1}$ );

$S_s$  is the specific storage defined as the ratio of the volume of water which can be injected per unit volume of aquifer material per unit change in head ( $L^{-1}$ )

$\Delta V$  is the volume of the cell ( $L^3$ ); and  $\Delta h$  is the change in head over a time interval,  $\Delta t$ . The term on the right hand side is equivalent to the volume of water taken into storage over a time interval  $\Delta t$  given a change in head of  $\Delta h$ .

Figure 3.1 depicts a cell  $i, j, k$  and six adjacent aquifer cells  $i-1, j, k$ ;  $i+1, j, k$ ;  $i, j+1, k$ ;  $i, j-1, k$ ;  $i, j, k-1$ , and  $i, j, k+1$ . Flow into cell  $i, j, k$  in the row direction from cell  $i, j-1, k$  (Fig. 3.1), according to Darcy's law, is given by

$$q_{i,j-1/2,k} = KR_{i,j-1/2,k} \Delta c_i \Delta v_k \frac{h_{i,j-1,k} - h_{i,j,k}}{\Delta r_{j-1/2}} \quad (3.3)$$

where

$q_{i,j-1/2,k}$  is the volumetric fluid discharge through the face between cells  $i, j, k$  and  $i, j-1, k$  ( $L^3 t^{-1}$ );  $KR_{i,j-1/2,k}$  is the hydraulic conductivity along the row between nodes  $i, j, k$  and  $i, j-1, k$  ( $L$ ).

The index  $j-1/2$  is used to indicate the space between nodes (Fig. 3.2). It does not indicate a point exactly halfway between nodes. For example,  $KR_{i,j-1/2,k}$  represents hydraulic conductivity in the entire region between nodes  $i,j,k$  and  $i,j-1,k$ .

Similar expressions can be written for approximately the flow into or out of the cell through the remaining five faces, as:

$$q_{i,j+1/2,k} = KR_{i,j+1/2,k} \Delta c_i \Delta v_k \frac{h_{i,j+1,k} - h_{i,j,k}}{\Delta r_{j+1/2}} \quad (3.4)$$

$$q_{i+1/2,j,k} = KC_{i+1/2,j,k} \Delta r_j \Delta v_k \frac{h_{i+1,j,k} - h_{i,j,k}}{\Delta c_{i+1/2}} \quad (3.5)$$

$$q_{i-1/2,j,k} = KC_{i-1/2,j,k} \Delta r_j \Delta v_k \frac{h_{i-1,j,k} - h_{i,j,k}}{\Delta c_{i-1/2}} \quad (3.6)$$

$$q_{i,j,k+1/2} = KV_{i,j,k+1/2} \Delta r_j \Delta c_i \frac{h_{i,j,k+1} - h_{i,j,k}}{\Delta v_{k+1/2}} \quad (3.7)$$

$$q_{i,j,k-1/2} = KV_{i,j,k-1/2} \Delta r_j \Delta c_i \frac{h_{i,j,k-1} - h_{i,j,k}}{\Delta v_{k-1/2}} \quad (3.8)$$

Each of the equations 3.3 -3.8 expresses flow through a face of cell  $i,j,k$  in terms of heads, grid dimensions, and hydraulic conductivity. Grid dimensions and hydraulic conductivity remain constant throughout the solution process so that the notation can be simplified by combining the constants into a single constant, which multiplies head, called the "hydraulic conductance" or, more simply, the "conductance". For example

$$CR_{i,j-1/2,k} = KR_{i,j-1/2,k} \Delta c_i \Delta v_k / \Delta r_{j-1/2} \quad (3.9)$$

Where

$CR_{i,j-1/2,k}$  is the conductance in row  $i$  and layer  $k$  between nodes  $i,j-1,k$  and  $i,j,k$  ( $L^2t^{-1}$ ).

Conductance is the product of hydraulic conductivity and cross-sectional area of flow divided by the length of the flow path; in this case, the distance between the nodes.

Substituting this expression into equations 3.3 to 3.8 yields

$$q_{i,j-1/2,k} = CR_{i,j-1/2,k} (h_{i,j-1,k} - h_{i,j,k}) \quad (3.10)$$

$$q_{i,j+1/2,k} = CR_{i,j+1/2,k} (h_{i,j+1,k} - h_{i,j,k}) \quad (3.11)$$

$$q_{i-1/2,j,k} = CC_{i-1/2,j,k} (h_{i-1,j,k} - h_{i,j,k}) \quad (3.12)$$

$$q_{i+1/2,j,k} = CC_{i+1/2,j,k} (h_{i+1,j,k} - h_{i,j,k}) \quad (3.13)$$

$$q_{i,j,k-1/2} = CV_{i,j,k-1/2} (h_{i,j,k-1} - h_{i,j,k}) \quad (3.14)$$

$$q_{i,j,k+1/2} = CV_{i,j,k+1/2} (h_{i,j,k+1} - h_{i,j,k}) \quad (3.15)$$

The above equations represent flow between the cells. The external flow into or out of the cells, such as river recharge or well pumpage can be expressed as

$$a_{i,j,k,n} = p_{i,j,k,n} h_{i,j,k} + q_{i,j,k,n} \quad (3.16)$$

Where

$a_{i,j,k,n}$  represents flow from the  $n$ -th external source into cell  $i,j,k$  ( $L^3t^{-1}$ ), and  $p_{i,j,k,n}$  and  $q_{i,j,k,n}$  are constants ( $L^2t^{-1}$  and  $L^3t^{-1}$ , respectively).



Substitution of equations 3.10 to 3.16 into equation 3.2 yields,

$$\begin{aligned}
 & CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) + CR_{i,j+1/2,k}(h_{i,j+1,k} - h_{i,j,k}) + \\
 & CC_{i-1/2,j,k}(h_{i-1,j,k} - h_{i,j,k}) + CC_{i+1/2,j,k}(h_{i+1,j,k} - h_{i,j,k}) + \\
 & CV_{i,j,k-1/2}(h_{i,j,k-1} - h_{i,j,k}) + CV_{i,j,k+1/2}(h_{i,j,k+1} - h_{i,j,k}) + \\
 & P_{i,j,k} h_{i,j,k} + Q_{i,j,k} = SS_{i,j,k} (\Delta r_j \Delta c_i \Delta v_k) \Delta h_{i,j,k} / \Delta t \quad (3.17)
 \end{aligned}$$

where  $Q_{i,j,k}$  is the sum of all  $q_{i,j,k,n}$  from different external sources and  $P_{i,j,k}$  is the sum of all components  $P_{i,j,k,n}$ . Using backward differencing for time, we get

$$\begin{aligned}
 & CR_{i,j-1/2,k}(h_{i,j-1,k}^m - h_{i,j,k}^m) + CR_{i,j+1/2,k}(h_{i,j+1,k}^m - h_{i,j,k}^m) + \\
 & CC_{i-1/2,j,k}(h_{i-1,j,k}^m - h_{i,j,k}^m) + CC_{i+1/2,j,k}(h_{i+1,j,k}^m - h_{i,j,k}^m) + \\
 & CV_{i,j,k-1/2}(h_{i,j,k-1}^m - h_{i,j,k}^m) + CV_{i,j,k+1/2}(h_{i,j,k+1}^m - h_{i,j,k}^m) + \\
 & P_{i,j,k} h_{i,j,k}^m + Q_{i,j,k} = SS_{i,j,k} (\Delta r_j \Delta c_i \Delta v_k) \frac{(h_{i,j,k}^m - h_{i,j,k}^{m-1})}{t_m - t_{m-1}} \quad (3.18)
 \end{aligned}$$

where  $h_{i,j,k}^m$  is the head at the end of time interval  $m$  and  $h_{i,j,k}^{m-1}$  is the known head at the end of the previous time interval  $m-1$ . For example, if we apply equation 3.18 for time interval 2, we get

$$\begin{aligned}
 & CR_{i,j-1/2,k}(h_{i,j-1,k}^2 - h_{i,j,k}^2) + CR_{i,j+1/2,k}(h_{i,j+1,k}^2 - h_{i,j,k}^2) + \\
 & CC_{i-1/2,j,k}(h_{i-1,j,k}^2 - h_{i,j,k}^2) + CC_{i+1/2,j,k}(h_{i+1,j,k}^2 - h_{i,j,k}^2) + \\
 & CV_{i,j,k-1/2}(h_{i,j,k-1}^2 - h_{i,j,k}^2) + CV_{i,j,k+1/2}(h_{i,j,k+1}^2 - h_{i,j,k}^2) + \\
 & P_{i,j,k} h_{i,j,k}^2 + Q_{i,j,k} = SS_{i,j,k} (\Delta r_j \Delta c_i \Delta v_k) \frac{(h_{i,j,k}^2 - h_{i,j,k}^1)}{t_2 - t_1} \quad (3.19)
 \end{aligned}$$

Equation 3.18 can be written in the following form

$$\begin{aligned}
 & CV_{i,j,k-1/2} h_{i,j,k-1}^m + CC_{i-1/2,j,k} h_{i-1,j,k}^m + CR_{i,j-1/2,k} h_{i,j-1,k}^m \\
 & + (-CV_{i,j,k-1/2} - CC_{i-1/2,j,k} - CR_{i,j-1/2,k} - CR_{i,j+1/2,k} \\
 & - CC_{i+1/2,j,k} - CV_{i,j,k+1/2} + HCOF_{i,j,k}) h_{i,j,k}^m + CR_{i,j+1/2,k} h_{i,j+1,k}^m \\
 & + CC_{i+1/2,j,k} h_{i+1,j,k}^m + CV_{i,j,k+1/2} h_{i,j,k+1}^m = RHS_{i,j,k} \quad (3.20)
 \end{aligned}$$

where

$$\begin{aligned}
 HCOF_{i,j,k} &= P_{i,j,k} - SC1_{i,j,k} / (t_m - t_{m-1}) \quad (L^2 t^{-1}) \\
 RHS_{i,j,k} &= -Q_{i,j,k} - SC1_{i,j,k} h_{i,j,k}^{m-1} / (t_m - t_{m-1}); \text{ and } (L^3 t^{-1}) \\
 SC1_{i,j,k} &= SS_{i,j,k} \Delta r_j \Delta c_i \Delta v_k \quad (L^2) \quad (3.21)
 \end{aligned}$$

Equation 3.20 is applied to each cell in the domain. We then get a set of linear algebraic simultaneous equations which should be solved at each time step. These simultaneous equations are usually solved by iterative methods.

### 3.2 PROGRAM DESIGN:

The computer program to simulate Three Dimensional Finite Difference Groundwater model was developed by the Water Resources Divison, Ministry of Interior of the United States Geological Survey (Mc Donald and Harabaugh, 1984). The following description closely follows the above reference. This model combines all the requirements for the evaluation of groundwater flow. The components like rainfall recharge, river recharge/discharge, well recharge/discharge, groundwater evapotranspiration, general head flow boundaries are all included in the model. Provison has also been made for inclusion of several other new options without affecting the overall structure of the program. The basic structure

of the program consists of a main program and a number of highly independent modules which perform the individual tasks for which they are meant. The main program invokes a particular module based on its necessity to compute the requirements provided in that module. The unwanted modules will not be utilized while simulating the model. Figure 3.3 shows the organization of the simulation.

The overall program structure with brief explanation is given in figure 3.4. The main program consists of a number of procedures namely DEFINE, ALLOCATE, READ and PREPARE, etc. Each simulation has three nested loops : a STRESS PERIOD LOOP, within which there is a TIME STEP LOOP and which in turn consists of an ITERATION LOOP.

In the DEFINE Procedure, the problem to be simulated is defined specifying the size of the model, the type of simulation (transient or steady state), the number of stress periods, hydrologic options and the solution scheme desired. In the ALLOCATE procedure, memory space required by the program is allocated. In the READ AND PREPARE procedure, all the data that are not functions of time are read including boundary conditions, initial heads, transmissivity/hydraulic conductivity, specific yield/storage coefficient, elevation of layer top and bottom and parameters of the specified solution scheme. Each package consists of all modules associated with a particular hydrologic feature and a solution method. For example, the RIVER package, performs all the tasks needed for simulation of the river flow effect on the model, similarly other packages like WELL package, DRAIN package, RECHARGE package, etc., perform their respective tasks.

### 3.3 IMPLEMENTATION OF THE PROGRAM :

The computer program originally developed by US Geological Survey, Water Resources division was written in Fortran 66. It is implemented in the Micro Vax II system which has a fortran 77 compiler and also DEC - 1090 at IIT Kanpur. Some of the modifications made while implementing the program are given below.

1) The program listing given does not contain any file "OPEN" statement. The Micro VAX II system resulted in certain problems during the execution and debugging of the program and hence the same is included.

```
OPEN(UNIT= 1,FILE='INBAS.DAT',STATUS='OLD')
```

```
OPEN(UNIT=16,FILE='IOUT.DAT',STATUS='NEW')
```

2) As the accuracy of the model till sixteen digits is not necessary, the DOUBLE PRECISION statements are deleted so as to save memory and also to reduce the computational time considerably. To effect this change the string is modified as,

```
NLAY * NROW * NCOL
```

3) Data statements in Micro VAX II system do not require ( ' ) before and after end of data string and the same was modified.

The program as modified above was compiled without any compilation errors.

### 3.4 VALIDATION OF THE PROGRAM:

The users manual contains one set of test data with typical output. The program was executed with the same sample input data and the results obtained are the same as given in the manual. A summary of the results is presented in Table 3.1 for comparison.

Table 3.1 Comparison of Results of Test Data  
VOLUMETRIC BUDGET FOR ENTIRE MODEL AT END OF TIME STEP 1  
IN STRESS PERIOD 1

	REPORTED IN THE MANUAL	Micro VAX II	DEC 1090
<u>CUMULATIVE VOLUMES</u> L ** 3			
<u>IN:</u>			
STORAGE	= 0.00000E+00	0.00000E+00	0.00000E+00
CONSTANT HEAD	= 0.00000E+00	0.00000E+00	0.00000E+00
WELLS	= 0.00000E+00	0.00000E+00	0.00000E+00
DRAINS	= 0.00000E+00	0.00000E+00	0.00000E+00
RECHARGE	= 0.13608E+08	0.13608E+08	0.13608E+08
TOTAL IN	= 0.13608E+08	0.13608E+08	0.13608E+08
<u>OUT:</u>			
STORAGE	= 0.00000E+00	0.00000E+00	0.00000E+00
CONSTANT HEAD	= 0.43265E+07	0.43265E+07	0.43265E+07
WELLS	= 0.64800E+07	0.64800E+07	0.64800E+07
DRAINS	= 0.28011E+07	0.28011E+07	0.28011E+07
RECHARGE	= 0.00000E+00	0.00000E+00	0.00000E+00
TOTAL OUT	= 0.13608E+08	0.13608E+08	0.13608E+08
IN - OUT	= 184.00	395.00	403.63
PERCENT DISCREPANCY	= 0.00	0.00	0.00
<u>RATES FOR THIS TIME STEP</u> L ** 3/T			
<u>IN:</u>			
STORAGE	= 0.00000E+00	0.00000E+00	0.00000E+00
CONSTANT HEAD	= 0.00000E+00	0.00000E+00	0.00000E+00
WELLS	= 0.00000E+00	0.00000E+00	0.00000E+00
DRAINS	= 0.00000E+00	0.00000E+00	0.00000E+00
RECHARGE	= 157.50	157.50	157.50
TOTAL IN	= 157.50	157.50	157.50
<u>OUT:</u>			
STORAGE	= 0.00000E+00	0.00000E+00	0.00000E+00
CONSTANT HEAD	= 50.075	50.075	50.075
WELLS	= 75.000	75.000	75.000
DRAINS	= 32.420	32.420	32.420
RECHARGE	= 0.00000E+00	0.00000E+00	0.00000E+00
TOTAL OUT	= 157.50	157.50	157.50
IN - OUT	= 0.21210E-02	0.045776E+-02	0.46711E-02
PERCENT DISCREPANCY	= 0.00	0.00	0.00

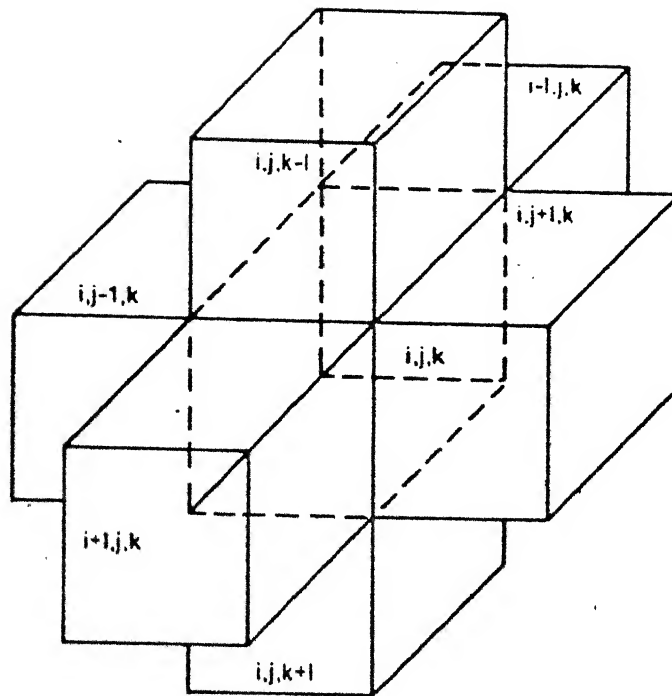


Figure 3] -Cell  $i,j,k$  and indices for the six adjacent cells.

(Adopted from Mc Donald and Harabaugh, 1984)

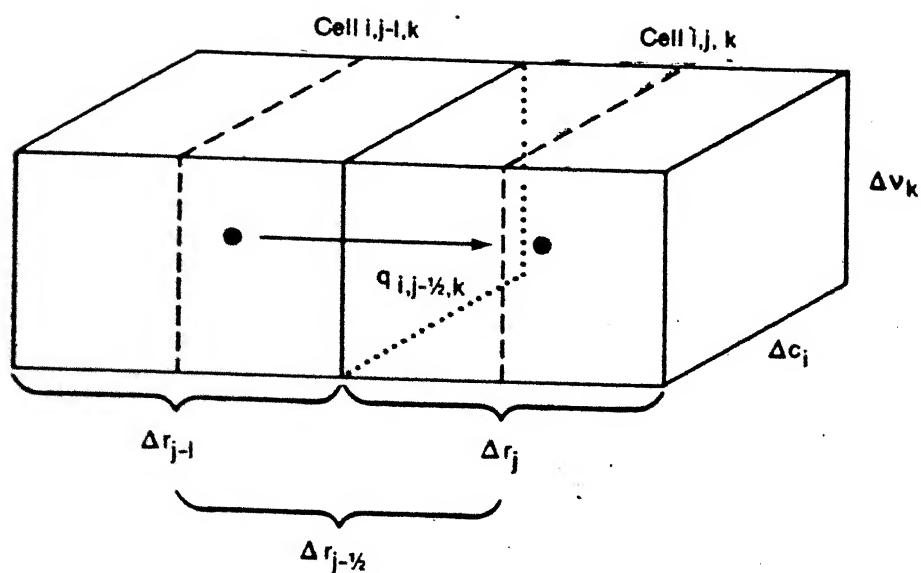


Figure 32.-Flow into cell i,j,k from cell i,j-1,k.

(Adopted from Mc Donald and Harabaugh, 1984)

Packages

	BAS	BCF	WEL	RCH	RIV	DRN	EVT	GHB	SIP	SOR
Define (DF)	BAS1DF									
Allocate (AL)	BAS1AL	BCF1AL	WEL1AL	RCH1AL	RIV1AL	DRN1AL	EVT1AL	GHB1AL	SIPIAL	SORIAL
Read & Prepare (RP)	BAS1RP <sub>U</sub>	BCF1RP <sub>US</sub>							SIP1RP	SOR1RP
Stress (ST)	BAS1ST									
Read & Prepare (RP)			WEL1RP	RCH1RP <sub>U</sub>	RIV1RP	DRN1RP	EVT1RP <sub>U</sub>	GHB1RP		
Advance (AD)	BAS1AD									
Formulate (FM)	BAS1FM	BCF1FM <sub>S</sub>	WEL1FM	RCH1FM	RIV1FM	DRN1FM	EVT1FM	GHB1FM		
Approximate (AP)									SIP1AP <sub>S</sub>	SOR1AP <sub>S</sub>
Output Control (OC)	BAS1OC									
Budget (BD)		BCF1BD <sub>US</sub>	WEL1BD <sub>U</sub>	RCH1BD <sub>U</sub>	RIV1BD <sub>U</sub>	DRN1BD <sub>U</sub>	EVT1BD <sub>U</sub>	GHB1BD <sub>U</sub>		
Output (OT)	BAS1OT <sub>U</sub>									

P R O C E D U R E S

Figure 3.3 Primary modules organized by procedure and package.

(Adopted from Mc Donald and Harabaugh, 1984)



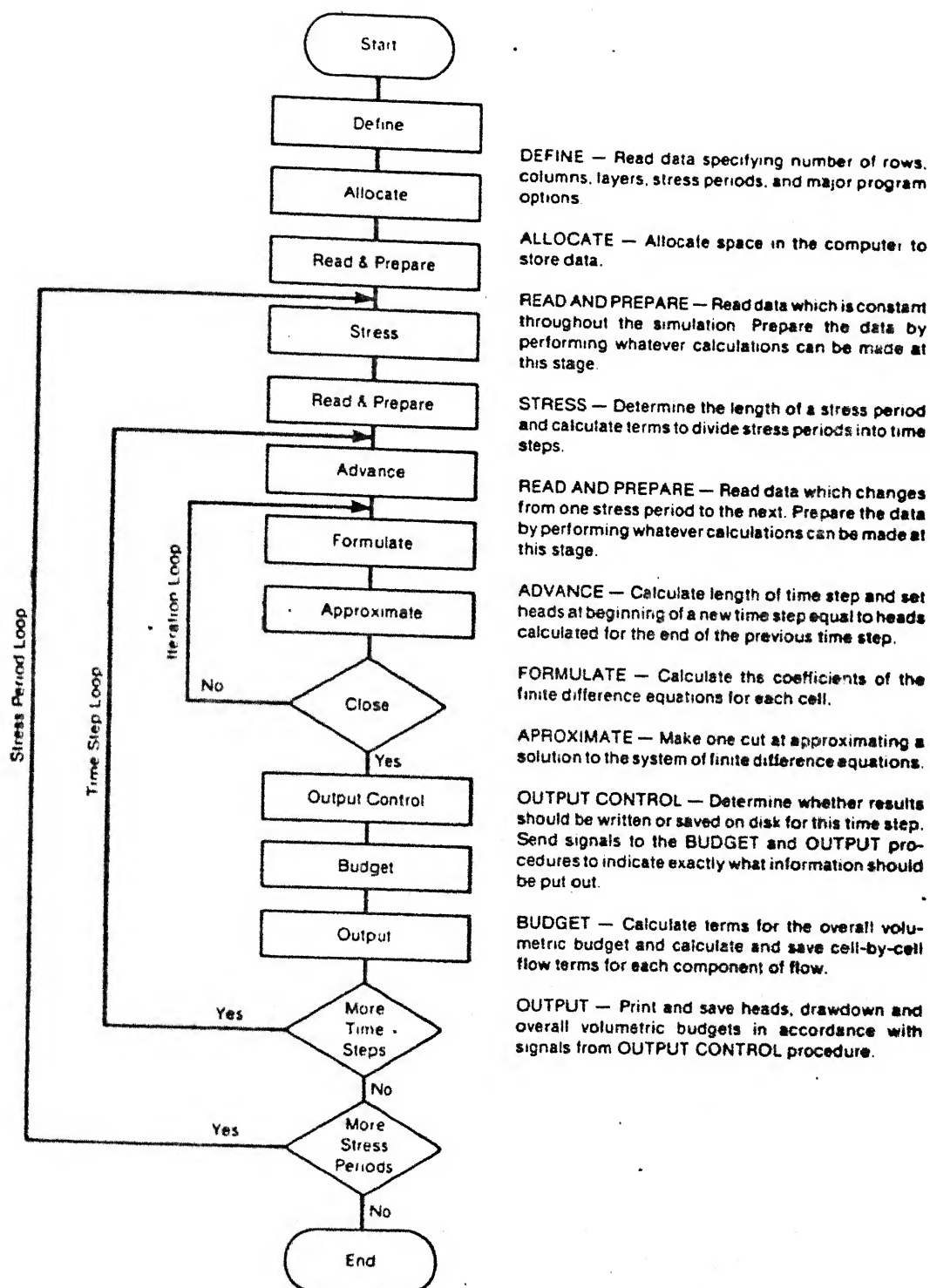


Figure 3.4-Overall program structure.

(Adopted from Mc Donald and Harabaugh, 1984)

## CHAPTER 4

### APPLICATION OF THE MODEL

#### 4.1 Study Area

The model described in the previous chapter is applied to a small catchment (or basin) namely Havaligi catchment in Anantapur District of Andhra Pradesh. The Havaligi catchment lies between latitude  $14^{\circ} 50'$  -  $15^{\circ} 05'$  and longitude  $77^{\circ} 05'$  -  $77^{\circ} 20'$  covering an area of  $267 \text{ km}^2$ .

The data for the above catchment were collected from Andhra Pradesh State Irrigation Development Corporation and Andhra Pradesh Ground Water Department.

Generally, for any groundwater studies we need data for atleast about five to ten years. However, in the present case data were available for only two years (1979 - 1980). The types of data collected are:

1. Topography of the area
2. Daily Rainfall
3. Daily evaporation (Pan)
4. Daily tank water levels and outflows
5. Canal length, width and its discharge
6. Borehole particulars
  - a) Lithology
  - b) Static water levels
  - c) Diameter of bore and depth drilled
7. Storage coefficient and transmissivity values
8. Command area particulars

Though we do not have agricultural data for the period of study, suitable assumptions based on present agricultural practices were made.

**4.1.1 Topography:** The catchment areas of Havaligi and Urvakonda tanks (anicut) are marked on the toposheets (Fig 4.1), delineating the catchment by joining the ridge lines. The streams and tanks present within each catchment area are also marked. The general topography can be easily seen from the Fig.4.1. Basically the catchment under study is plain with very flat slope.

**4.1.2 Rainfall Data:** There are fifteen raingauge stations in the Havaligi catchment. The rainfall data for the stations is given in Table 4.1. Though daily rainfall data for two years is available, only monthly values are given in Table 4.1. Some missing rainfall data were estimated by using the weighted average method.

**4.1.3 Pan Evaporation Data:** The daily pan evaporation data were recorded at the hydrological station in the basin using ISI pan evaporimeter which is similar to U.S. Class A pan both in construction and in dimensions. These values are represented by a graph (Fig. 4.2).

**4.1.4 Tank Water Levels:** There are two tanks in the catchment area. Daily water levels of the tanks are available and the contours of the tanks are also available, from which we can calculate the water spread area, which is useful for calculating evaporation losses from the tanks and seepage losses. Also, we have the daily outflows from the tanks through the sluices and over the surplus weirs. Figure 4.3 shows the contour map of

Havaligi tank and figure 4.4 shows the map of the Urvakonda tank. Figures 4.5 and 4.6 respectively show the stage versus water spread area and stage versus cumulative volumes of Havaligi and Urvakonda tanks.

**4.1.5 Canal Particulars:** Two major unlined canals pass through Havaligi catchment. These are Tungabhadra High Level Canal and Guntakal Branch Canal. These are shown in Fig. 4.1. For water balance study one would require the amount of water that enters the basin through canal and the amount of water that leaves the basin through the same canal. This would enable computation of canal losses over the basin. However, such information is not available for the present basin. The seepage losses from the canals are calculated by using Darcy's law and by making assumptions about canal bed material.

**4.1.6 Borehole Data:** Data on several boreholes in the catchment area were available. The lithological data collected while drilling wells are available. A typical lithology is given in Fig. 4.7.

**4.1.7 Command Area Particulars:** The command area of Urvakonda tank is presented in fig 4.8. The type of crops grown in each season were not available. However, assumptions have been made about these based on the crops grown in the basin presently. Table 4.2 shows the type of crops and their crop coefficients.

A groundwater balance study is required for evaluating groundwater resources. In the following sections two methods are presented. In the first method, the overall hydrological balance for the basin as a whole including both surface water and groundwater is presented. In this method, the spatial variation

within the basin in hydrological parameters as well as aquifer parameters are not taken into account. In the second method, a detailed three dimensional model is used which takes into account all the variations in the basin (Lakshminarayana 1989).

## 4.2 Hydrologic Budget Model

Groundwater is a resource which is being developed more and more. Groundwater storage, in contrast with surface storage, is unseen. There are several questions one has to consider in connection with groundwater development: How much water can be drawn safely? What happens if more water than this is drawn? How can we increase the supply to groundwater basin?

### 4.2.1 Hydrologic Balance

In order to determine how much water can be taken out of groundwater safely without causing undesirable effects, one has to do groundwater balance or groundwater budget study. A groundwater basin is a part of the total hydrologic basin which includes both surface water and subsurface water. The general form of the hydrologic balance includes a statement of all the waters entering and leaving the basin. It may be expressed as

$$\text{Inflow} - \text{Outflow} = \text{Change in storage}$$

The inflow includes all components such as precipitation, surface inflow, subsurface inflow, water imported from outside the basin, etc. The outflow includes all items such as surface outflow, subsurface outflow, consumptive use, water exported

outside the basin, etc. The change in storage includes change in surface water storage and change in subsurface storage. The change in subsurface storage should account for both soil moisture storage and groundwater storage.

Because of interrelation of surface and groundwater, all inflows and outflows of the basin must be included in the balance equation. Often certain items can be omitted because they are negligible. Any consistent unit of volume and time can be employed, say million cubic meters per year. The surface inflow, outflow, precipitation, imported and exported water are all measurable. The consumptive use or evapotranspiration can be estimated by determining the various types of crops grown and other types of vegetation such as forests in the basin. The sum of the products for each land use, area times consumptive use yields the total consumptive use.

The changes in the surface storage can be evaluated directly. The subsurface inflow and outflow are difficult to estimate. If it is possible to choose the basin such that no subsurface flow crosses into and out of the groundwater basin then these terms are zero. If sufficient data are available it would be possible to estimate them by Darcy's law.

By selecting time intervals in which the amount of water in unsaturated zone is nearly equal, only changes in saturated zone need be considered. From measurement of water level in wells, maps of groundwater changes can be prepared. The product of change in water level times specific yield times area gives change in groundwater storage.

#### 4.2.2 Groundwater Balance

A specialized form of hydrologic balance equation in which all items pertaining to a groundwater reservoir is called a groundwater balance or budget equation. Recharge is the main inflow component for a groundwater basin. If there are inflows and outflows across the boundary they can be estimated by using Darcy's law. The main outflows are groundwater runoff, groundwater evapotranspiration and groundwater pumpage. A groundwater balance equation can be written as

$$P_g - R_g - ET_g - W = \Delta H \cdot S_y$$

where  $P_g$  = groundwater recharge  
 $R_g$  = groundwater runoff  
 $W$  = groundwater pumpage (withdrawal)  
 $ET_g$  = groundwater evapotranspiration  
 $\Delta H$  = change in groundwater level  
 $S_y$  = Specific yield

Again consistent system of units should be used in the above equation.

#### 4.2.3 Groundwater Recharge

The main source of groundwater is precipitation which may infiltrate into the ground and finally reach groundwater table, or may enter surface streams and seep from these channels to the groundwater. Interception by vegetation and soil moisture must be satisfied before any large amount of rainwater can percolate into groundwater. Only prolonged period of heavy precipitation can supply large quantities of water for groundwater recharge.

Groundwater recharge is an intermittent and irregular process. Water used in irrigation in excess of requirement of crops may also become groundwater recharge. Groundwater levels are usually at their lowest in the month of May i.e., just before monsoon and will reach their highest level in the month of October, just after the monsoon in this country. Groundwater development must be made in a planned way. On a long term basis the amount of groundwater taken from a groundwater basin should not be more than the natural groundwater recharge reaching the aquifer.

#### 4.2.4 Groundwater Evapotranspiration

In areas where water table is shallow and the capillary fringe extends to the land surface, large quantities of groundwater are discharged into atmosphere by evaporation and transpiration by plants. This process is called evapotranspiration. Evapotranspiration depends primarily on meteorological factors, available soil moisture and groundwater, and type of soil and vegetation. At many places, evapotranspiration is small during winter months and reaches a maximum during summer months. In summer months evapotranspiration is very effective in reducing groundwater runoff. With the same mean groundwater stage groundwater runoff is much less in summer than in winter. Separate rating curves, that is groundwater stage versus groundwater runoff curves, must be prepared for summer and winter months. The difference in groundwater runoff between the two curves can often be used as a rough estimate of groundwater evapotranspiration.

The following inputs and outputs are computed for applying the hydrologic budget equation to the Havaligi catchment.



#### 4.2.5 Inputs

**4.2.5a Rainfall:** Since there is variation in the distribution of rainfall over the basin as seen from the rainfall data recorded at different raingauge stations, the basin is divided into a number of Thiessen polygons (Fig 4.9) and the rainfall within each of the Thiessen polygons is assumed to be the same as recorded by the raingauge located within the polygon. This method is justified because the basin is not located in the mountaneous region and the areas of each of the Thiessen polygons is small (3% to 14% of the total catchment area). The total volume of the rainfall contribution is determined by

$$I_p = A_1P_1 + A_2P_2 + \dots + A_nP_n$$

These results are shown in Table. 4.3 and the area of each of the Thiessen polygon are shown in Table 4.4

**4.2.5b Seepage from Canals and Tanks:** The wetted area of the tanks are taken as the areas of water spread. For the tanks, the water spread area is computed as the mean of maximum and minimum water spread areas in each month. The water spread areas are obtained from the stage vs. water spread area curves (Fig 4.5 and 4.6). Assuming the vertical hydraulic conductivity of the tank bed and the thickness of the bed material, the recharge to the groundwater is evaluated, using Darcy's law. They are shown in the Table 4.5.

**4.2.5.c Surface and subsurface inflows:** If there are any surface and subsurface inflows into the study area, they are also to be considered for evaluation. The only surface inflow into the study area is through canal and it is already considered in the previous section. Due to lack of sufficient information about the subsurface inflows it is assumed that the subsurface boundary is congruent with the surface boundary of the area. Hence the subsurface inflows are absent.

#### 4.2.6. Outputs

Not all the precipitated water reaches the groundwater. There are losses like Evaporation, Evapotranspiration, surface runoff etc.

**4.2.6a Evaporation and Evapotranspiration:** Based on the pan Evaporation values, the reference evapotranspiration and potential evapotranspiration can be calculated

$$ET_{\emptyset} = E * K_p$$

$$PET = ET_{\emptyset} * K_c$$

where  $K_p$  is pan coefficient.

$K_c$  is crop coefficient.

$ET_{\emptyset}$  is reference evapotranspiration.

PET is potential evapotranspiration.

E is pan evaporation.

The values of  $K_p$  and  $K_c$  are taken from FAO Irrigation and Drainage paper 24 (1975) and FAO Irrigation and Drainage paper 33 (1978) and the calculated values of the PET are given in Table 4.6.

**4.2.6b Surface Runoff:** The surface outflows from the catchment area are available and are shown in Table 4.7.

#### **4.2.7 The Hydrologic Budget:**

Based upon the above mentioned inputs and outputs for each of the month, the yearly hydrologic budget is made to determine the groundwater recharge. Due to lack of information about the soil moisture, the same was not included in the balance equation. However, this will not affect the yearly hydrologic budget, since it is assumed that the soil moisture will be same for the same seasons of different years. The results are discussed in Chapter 5.

#### **4.3 The Three Dimensional Finite Difference Model:**

In the previous section we have used hydrologic balance for the basin as a whole and obtained the total volumetric budget per annum. In the present section we will include the variations within the basin from point to point. For this it is necessary to use three dimensional finite difference approach. The basic equations for this method have already been given in Chapter 3.

##### **4.3.1 Discretization of the basin:**

In order to apply the finite-difference method, the basin was discretized into rows, columns and layers. The basin under present study is Havaligi basin. This basin has been discretized into 31 rows, 40 columns and 4 layers (Fig 4.10). The space between the rows and columns is same forming a square block of each layer which is same throughout that layer and it differs from other layers. These layers are the actual thicknesses of each of the geological formation present in the basin, which was taken from borehole data.

#### 4.3.2 Boundary Conditions:

The boundary conditions are applied to the basin. The boundary conditions considered are of three types and are specified by codes which were referred as

IBOUND < 0 constant head cell.

IBOUND = 0 inactive cell.

IBOUND > 0 variable head cell.

The constant head cell is one in which the variation of head is absent. And the variable head cell is the one in which the groundwater head changes in conjunction with the various inputs, outputs and interaction with boundaries. The discretized basin with boundary conditions are shown in fig 4.11.

#### 4.3.3 Initial Conditions

Though data for each cell is not available, but sufficient number of well data could overcome this difficulty. Based on the available information of static water levels in the wells, the water table contours were drawn fig 4.12 and the same data is given as input for initial heads in each of the cells.

#### 4.3.4 Rainfall Recharge

The net effective rainfall recharge is computed based on the available data like rainfall, evapotranspiration and the surface runoff and the same is given as effective recharge to each of the cells. They are presented in Table 4.8.

#### 4.3.5 Canal and Tank Recharge:

There are two canals passing through the basin under study and the cells through which they pass are shown in the Fig 4.1. The data required for the evaluation of canal seepage is shown in Table 4.9. Though the program does not have provision for computing the tank recharges/discharges, we have utilized the canal package for this evaluation because the mathematical formulation for both cases is the same.

#### 4.3.6 Well Pumping:

The location of wells and their particulars such as depth drilled, quantities of water pumped for each time period are given as the input for evaluation Table 4.10.

#### 4.4 Application of the model:

For the detailed evaluation of the Havaligi basin using three dimensional finite difference method, the study area is discretized as explained earlier. Considering a time step of one month in each stress period the simulation is carried out using Slice Successive Over Relaxation (SSOR) solver package using the above mentioned data. The volumetric budget for each stress period and the head in each cell for each layer are estimated. The drawdown in each cell and in each layer are also obtained. The results are discussed in Chapter 5.

TABLE 4.1 MONTHLY RAINFALL DATA OF HAVALIGI CATCHMENT (mm)

Rq.No	YEAR 1979											
	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	7.6000	35.4000	26.6000	58.0000	67.0000	14.0000	75.7000	0.0000
2	0.0000	0.0000	0.0000	0.0000	7.7000	35.4000	27.8000	48.0000	144.6000	13.7000	39.8000	0.0000
3	0.0000	0.0000	0.0000	0.0000	7.8000	34.3000	23.7000	34.5000	142.6000	18.5000	37.5000	0.0000
4	0.0000	0.0000	0.0000	0.0000	10.1000	29.1000	43.0000	27.8000	150.2000	14.4000	35.3000	0.0000
5	0.0000	0.0000	0.0000	0.0000	8.9000	50.4000	39.1000	52.3000	164.2000	9.9000	35.9000	0.0000
6	0.0000	0.0000	0.0000	0.0000	14.0000	26.5000	27.6000	23.5000	226.3000	37.5000	22.1000	0.0000
7	0.0000	0.0000	0.0000	0.0000	13.5000	37.5000	27.7000	48.1000	113.0000	23.9000	30.6000	0.0000
8	0.0000	0.0000	0.0000	0.0000	11.2000	39.2000	29.7000	28.1000	142.8000	28.5000	28.9000	0.0000
9	0.0000	0.0000	0.0000	0.0000	13.0000	40.1000	19.8000	56.6000	180.4000	30.1000	50.9000	0.0000
10	0.0000	0.0000	0.0000	0.0000	10.8000	31.3000	24.1000	74.6000	184.6000	57.2000	62.3000	0.0000
11	0.0000	0.0000	0.0000	0.0000	10.8000	31.4000	25.9000	130.1000	185.6000	61.0000	61.2000	0.0000
12	0.0000	0.0000	0.0000	0.0000	13.8000	38.3000	25.5000	150.1000	189.1000	69.0000	64.1000	0.0000
13	0.0000	0.0000	0.0000	0.0000	13.8000	38.0000	25.1000	167.1000	217.0000	69.8000	67.3000	0.0000
14	0.0000	0.0000	0.0000	0.0000	18.6000	28.2000	22.8000	153.2000	177.4000	43.2000	69.2000	0.0000
15	0.0000	0.0000	0.0000	0.0000	16.8000	38.3000	25.6000	101.8000	173.5000	55.9000	61.9000	0.0000

Rq.No	YEAR 1980											
	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	15.1000	40.6400	10.1500	15.4800	166.8000	27.5000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	16.2000	40.2000	16.2500	20.5600	178.1000	26.5000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	15.8000	74.5100	7.6200	12.4300	161.4500	28.2000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	14.9000	37.6000	12.7000	20.3000	173.0000	26.9000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	17.1000	40.9000	15.5000	21.3000	158.5000	28.1000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	16.5000	36.2000	9.9000	16.5000	148.3000	28.3000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	16.9000	29.4500	13.4600	23.3500	186.3600	29.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	15.4000	41.8000	22.5000	52.6000	162.0000	28.7000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	17.8000	40.9000	23.6000	53.0000	160.1000	27.7000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	16.0000	23.6000	15.5000	28.5000	173.0500	27.5000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	15.6000	24.9000	16.2000	28.3000	177.9000	27.7000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	15.9000	25.9000	17.0000	30.4000	168.3500	27.5000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	16.3000	24.1000	13.9000	34.1000	164.5000	27.3000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	16.2000	50.0000	19.0000	34.5000	178.6000	34.1000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	17.9000	42.6000	17.3000	36.6000	150.2000	32.5000	0.0000	0.0000

SOURCE A.P.STATE IRRIGATION DEVELOPMENT CORPORATION

Table 4.2 Types of crops and their crop coefficients

CROP	INITIAL STAGE	DEVELOPMENT STAGE	MID SEASON	LATE SEASON	TOTAL PERIOD
RICE	1.15	1.50	1.30	1.05	1.20
→	30	30	45	55	160
GROUNDNUTS	0.50	0.80	1.10	0.85	0.80
→	25	35	45	25	130
MILLETS	0.40	0.75	1.15	0.80	0.85
→	15	25	40	25	105
COTTON	0.50	0.80	1.25	0.90	0.90
→	30	50	60	55	195

→ represent length of each growth period in days.

TABLE 4.3 MONTHLY RAINFALL VOLUMES IN EACH THIESSEN POLYGON ( Million Cubic metres )

POLYGON No	YEAR 1979											
	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	0.1846	0.8600	0.6462	1.4091	1.6277	0.3401	1.8391	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.2829	1.3007	1.0214	1.7636	5.3129	0.5034	1.4623	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.2526	1.1106	0.7674	1.1171	4.6173	0.5990	1.2142	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.3594	1.0354	1.5300	0.9892	5.3443	0.5124	1.2560	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.1065	0.6031	0.4679	0.6259	1.9650	0.1185	0.4296	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.1547	0.2927	0.3049	0.2596	2.4999	0.4143	0.2441	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.2369	0.6581	0.4862	0.8442	1.9832	0.4195	0.5370	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.1470	0.5146	0.3899	0.3689	1.8746	0.3741	0.3794	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.2029	0.6259	0.3091	0.8835	2.8159	0.4698	0.7945	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.1476	0.4278	0.3294	1.0196	2.5231	0.7818	0.8515	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.1396	0.4059	0.3348	1.6819	2.3994	0.7886	0.7912	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.1085	0.3012	0.2005	1.1805	1.4872	0.5427	0.5041	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.1674	0.4608	0.3044	2.0265	2.6316	0.8465	0.8162	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.2702	0.4097	0.3313	2.2258	2.5774	0.6276	1.0054	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.1311	0.2989	0.1998	0.7945	1.3541	0.4363	0.4831	0.0000

YEAR 1980

POLYGON No	YEAR 1980														
	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER	JANUARY	FEBRUARY	MARCH
1	0.0000	0.0000	0.0000	0.0000	0.3668	0.9873	0.2466	0.3761	4.0523	0.6681	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.5952	1.4770	0.5971	0.7554	6.5438	0.9737	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.5116	2.4126	0.2467	0.4025	5.2277	0.9131	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.5302	1.3379	0.4519	0.7223	6.1556	0.9571	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.2046	0.4895	0.1855	0.2549	1.8968	0.3363	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.1823	0.3999	0.1094	0.1823	1.6382	0.3126	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.2966	0.5169	0.2362	0.4098	3.2707	0.5090	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.2022	0.5487	0.2954	0.6905	2.1267	0.3768	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.2778	0.6384	0.3684	0.8273	2.4990	0.4324	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.2187	0.3226	0.2119	0.3895	2.3653	0.3759	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.2017	0.3219	0.2094	0.3659	2.2998	0.3581	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.1250	0.2037	0.1337	0.2391	1.3240	0.2163	0.0000	0.0000	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.1977	0.2923	0.1686	0.4135	1.9949	0.3311	0.0000	0.0000	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.2354	0.7264	0.2760	0.5012	2.5948	0.4954	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.1397	0.3325	0.1350	0.2857	1.1723	0.2537	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 4.4 AREAS OF THE THIESSEN POLYGONS ( Sq kms )

POLYGON No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
24.295	36.742	32.380	35.581	11.967	11.047	17.551	13.128	15.609	13.668	12.928	7.865	12.127	14.529	7.805	



TABLE 4.5 MONTHLY SEEPAGES FROM TANKS AND CANALS ( Million Cubic Metres )

	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
	YEAR 1979											
TANKS	0.9949	0.8453	0.0000	0.0000	0.0000	0.6847	0.2858	1.5897	2.9489	2.6750	2.6033	0.7388
CANALS	0.9695	0.0000	0.0000	0.0000	0.0000	0.0000	0.4491	0.6572	1.1089	1.0826	1.1089	1.1089
	YEAR 1980											
TANKS	0.6533	0.2470	0.0203	0.0000	0.0000	0.0993	1.2532	1.3795	2.0511	0.0000	0.0000	0.0000
CANALS	0.9695	0.9695	0.0000	0.0000	0.0000	0.0000	0.7358	0.9826	1.0439	0.9695	0.9565	1.0686

**TABLE 4.6**  
**POTENTIAL EVAPOTRANSPIRATION VALUES**

Month	1979 (million cubic meters)	1980
January	1.3073	1.4488
February	1.3302	0.8983
March	0.0000	0.0529
April	0.0000	0.0000
May	0.0000	0.0000
June	5.4554	5.4505
July	6.9338	5.6650
August	5.8690	3.5010
September	7.3740	9.6124
October	4.7453	4.8105
November	1.8185	0.8013
December	1.4499	0.7270

**TABLE 4.7**  
**OUTFLOWS FROM HAVALIGI TANK**

Month	1979 (million cubic meters)	1980
January	0.0419	0.0414
Febrary	0.0011	0.0036
March	0.0000	0.0000
April	0.0000	0.0000
May	0.0000	0.0000
June	0.0000	0.0000
July	0.0000	0.0000
August	0.0692	0.0736
September	0.0215	0.0181
October	0.1428	0.0000
November	0.1464	0.0000
December	0.1219	0.0000

TABLE 4.8 EFFECTIVE RAINFALL RECHARGE TO THE GROUNDWATER

YEAR 1979												
Rg. No.	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	0.0018	0.1806	0.0000	0.6772	0.4324	0.0000	1.0392	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0064	0.3559	0.0000	0.7225	3.5825	0.0000	0.3163	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0091	0.2595	0.0000	0.1831	3.0733	0.0000	0.1884	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0916	0.1154	0.3744	0.0000	3.6638	0.0000	0.1423	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0165	0.1867	0.0112	0.2000	1.2993	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0715	0.0000	0.0000	0.0000	1.8738	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.1049	0.1227	0.0000	0.2796	1.0775	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0483	0.0736	0.0000	0.0000	1.1588	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0856	0.1321	0.0000	0.3668	1.9933	0.0000	0.2355	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0449	0.0000	0.0000	0.5510	1.7838	0.1168	0.3464	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0424	0.0000	0.0000	1.2319	1.6921	0.1508	0.3068	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0495	0.0000	0.0000	0.8562	0.9974	0.0873	0.1605	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0763	0.0414	0.0000	1.5959	1.9583	0.2368	0.3537	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.1612	0.0000	0.0000	1.7348	1.8002	0.0000	0.4755	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0727	0.0000	0.0000	0.4707	0.8661	0.0000	0.1403	0.0000

YEAR 1980												
Rg.No	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.3900	0.0000	0.0000	2.5606	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.6128	0.0000	0.0000	4.4060	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000	1.6485	0.0000	0.0000	3.2911	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.4985	0.0000	0.0000	4.0780	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.1577	0.0000	0.0000	1.0404	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0873	0.0000	0.0000	0.8316	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0652	0.0000	0.0000	2.1251	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.1936	0.0000	0.2677	1.2049	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.2305	0.0000	0.3632	1.4466	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.4099	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.3885	0.0000	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6734	0.0000	0.0000	0.0000
13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	1.1168	0.0000	0.0000	0.0000
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.3475	0.0000	0.0312	1.5737	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0968	0.0000	0.0000	0.5019	0.0000	0.0000	0.0000

All Values are in Million Cubic Metres

All Values are in Million Cubic Metres

TABLE 4.9 CANAL PARTICULARS (JANUARY 1979)

Layer	Row	Column	Stage (m)	Conductance ( $m^3/month$ )	Bottom Elevation(bgl)*
2	19	7	1.7	365.60	3.0
2	19	8	1.7	365.60	3.0
2	18	9	1.7	365.60	3.0
2	18	10	1.7	365.60	3.0
2	18	11	1.7	365.60	3.0
2	18	12	1.7	365.60	3.0
2	18	13	1.7	365.60	3.0
2	19	14	1.7	365.60	3.0
2	19	15	1.7	365.60	3.0
2	20	16	1.7	365.60	3.0
2	21	17	1.7	365.60	3.0
2	21	18	1.7	365.60	3.0
2	22	19	1.7	365.60	3.0
2	23	20	1.7	365.60	3.0
2	24	21	1.7	365.60	3.0
2	24	22	1.7	365.60	3.0
2	24	23	1.7	365.60	3.0
2	24	24	1.7	365.60	3.0
2	24	25	1.7	365.60	3.0
2	24	26	1.7	365.60	3.0
2	24	27	1.7	365.60	3.0
2	24	28	1.7	365.60	3.0
2	23	21	1.7	268.70	3.0
2	22	22	1.8	268.70	3.0
2	21	22	1.8	268.70	3.0
2	20	22	1.8	268.70	3.0
2	19	22	1.8	268.70	3.0
2	18	22	1.8	268.70	3.0
2	17	23	1.8	268.70	3.0
2	16	23	1.8	268.70	3.0
2	15	23	1.8	268.70	3.0
2	14	22	0.9	234.20	3.0
2	13	22	0.9	234.20	3.0
2	13	21	0.9	234.20	3.0
2	12	20	0.9	234.20	3.0
2	11	19	0.9	234.20	3.0
2	11	18	0.9	234.20	3.0
2	10	18	0.9	234.20	3.0
2	9	17	0.9	234.20	3.0
2	8	17	0.9	234.20	3.0
2	7	17	0.9	234.20	3.0
2	6	17	0.9	234.20	3.0

\* bgl below ground level

Table 4.10 Wells Location and Discharges  
for January 1979

JANUARY 1979			
LAYER	ROW	COLUMN	DISCHARGE (m <sup>3</sup> /day)
3	2	11	50.0
3	5	12	37.5
3	6	27	50.0
3	7	8	60.0
3	8	15	45.0
3	8	20	60.0
3	8	33	50.0
3	9	24	50.0
3	9	29	50.0
3	10	38	50.0
3	11	17	52.5
3	11	26	50.0
3	12	6	50.0
3	14	23	50.0
3	14	37	50.0
3	15	12	75.0
3	16	15	60.0
3	16	20	50.0
3	16	24	50.0
3	16	32	50.0
3	17	35	50.0
3	18	19	50.0
3	18	29	75.0
3	20	13	50.0
3	20	24	90.0
3	21	28	50.0
3	22	15	50.0
3	23	24	90.0
3	24	19	50.0
3	25	22	50.0
3	26	26	50.0



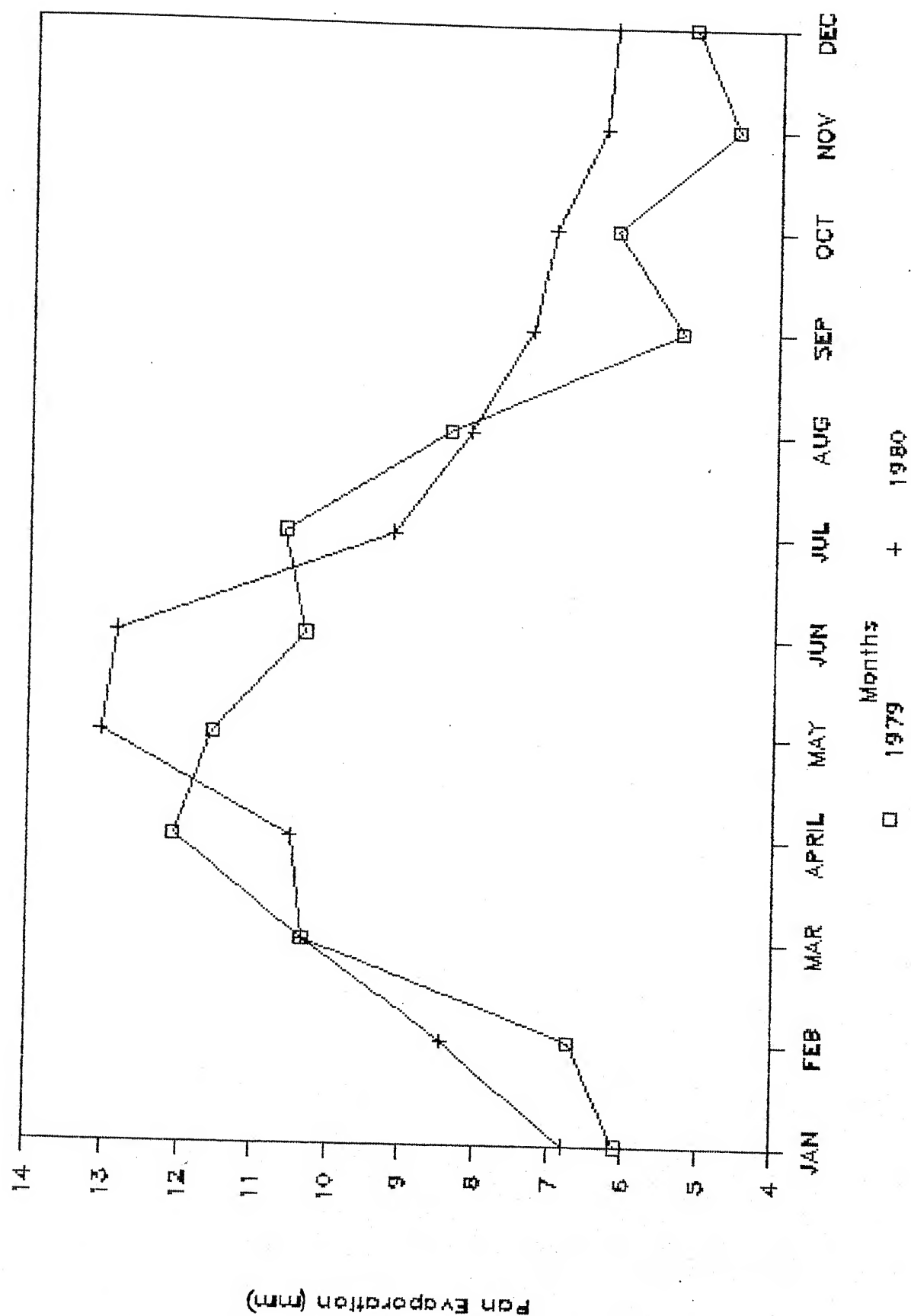


Fig. 4.2 Daily Pan Evaporation



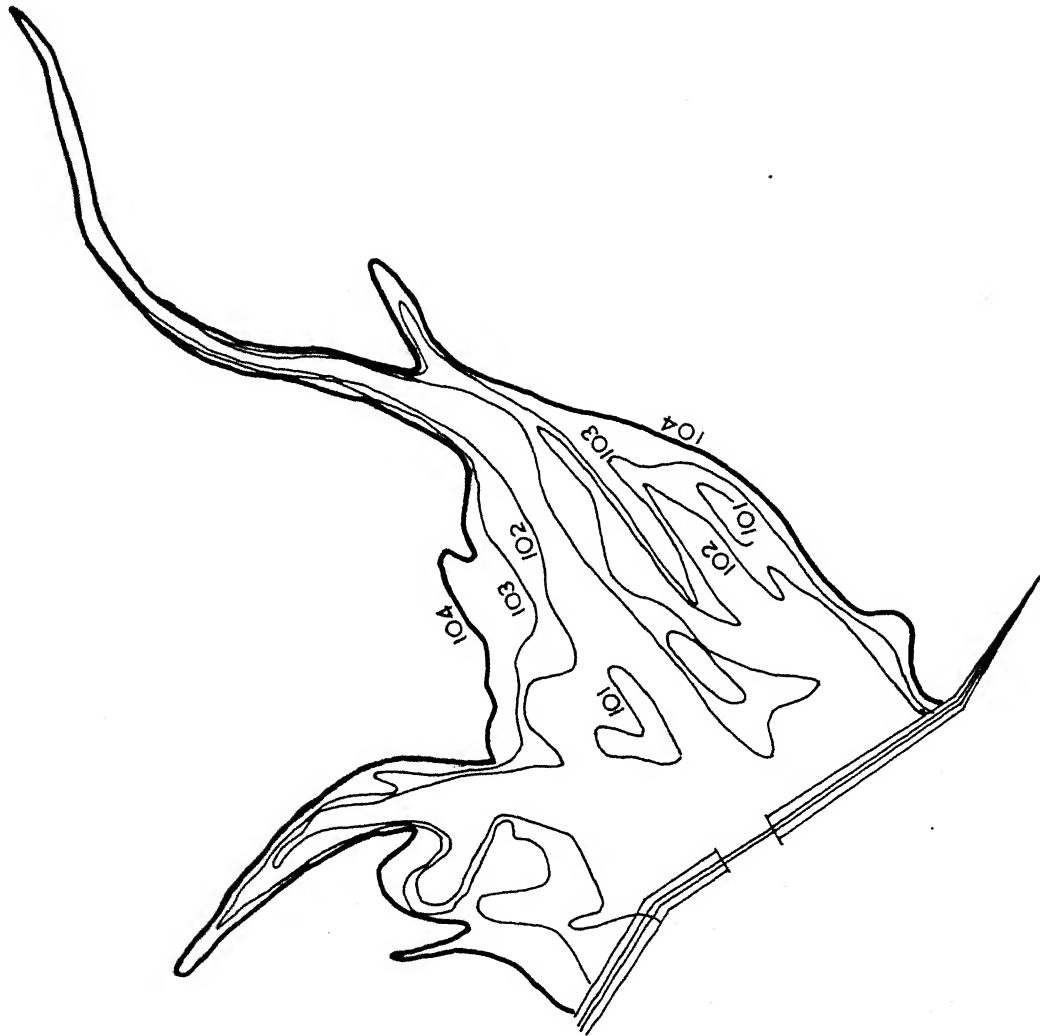


FIG.4.3 CONTOUR MAP OF HAVALIGI TANK

Acc. No. 106306



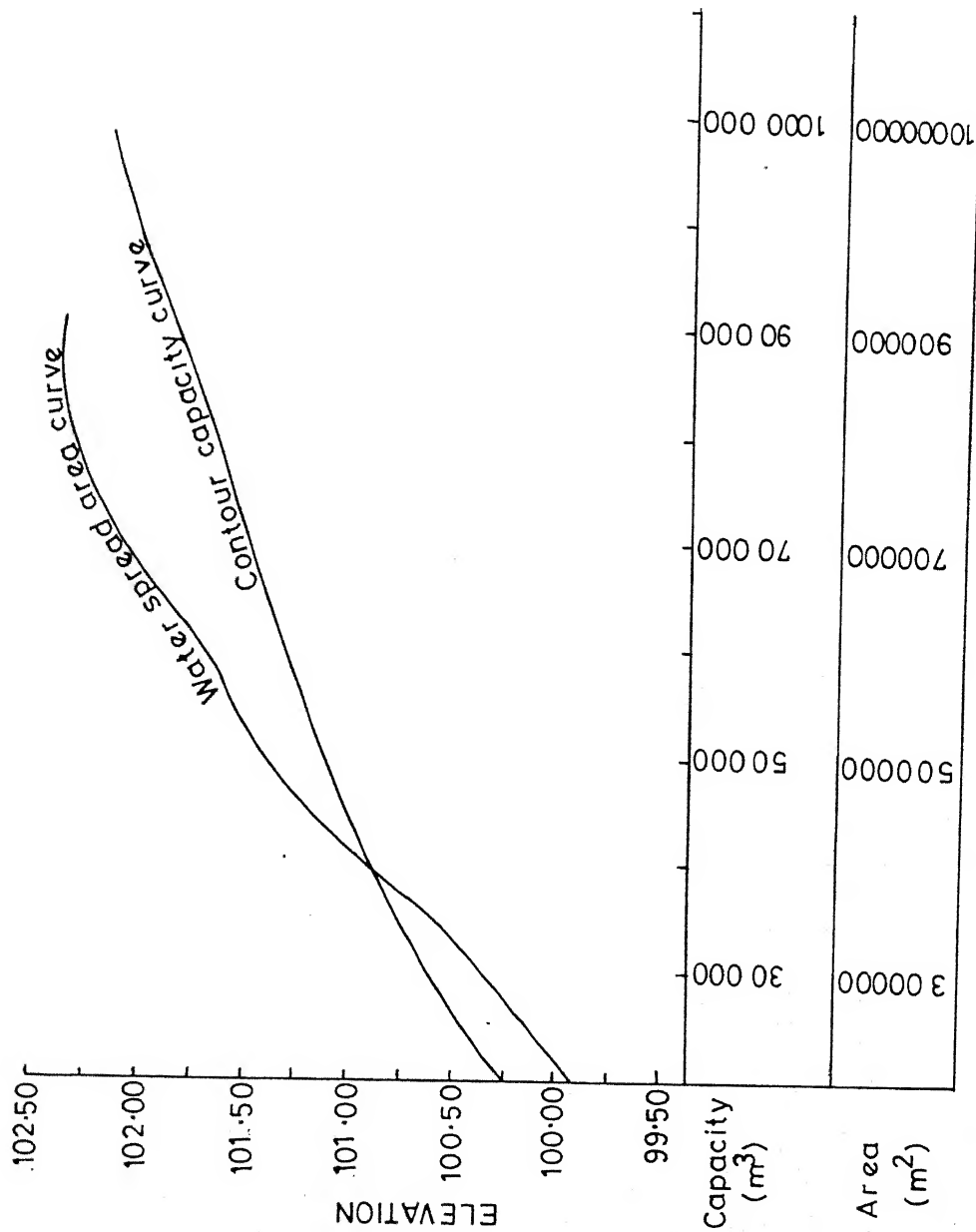


FIG.4.6 HAVALIGI TANK PARTICULARS  
(Stage Vs area and Stage Vs Cumulative Volume)

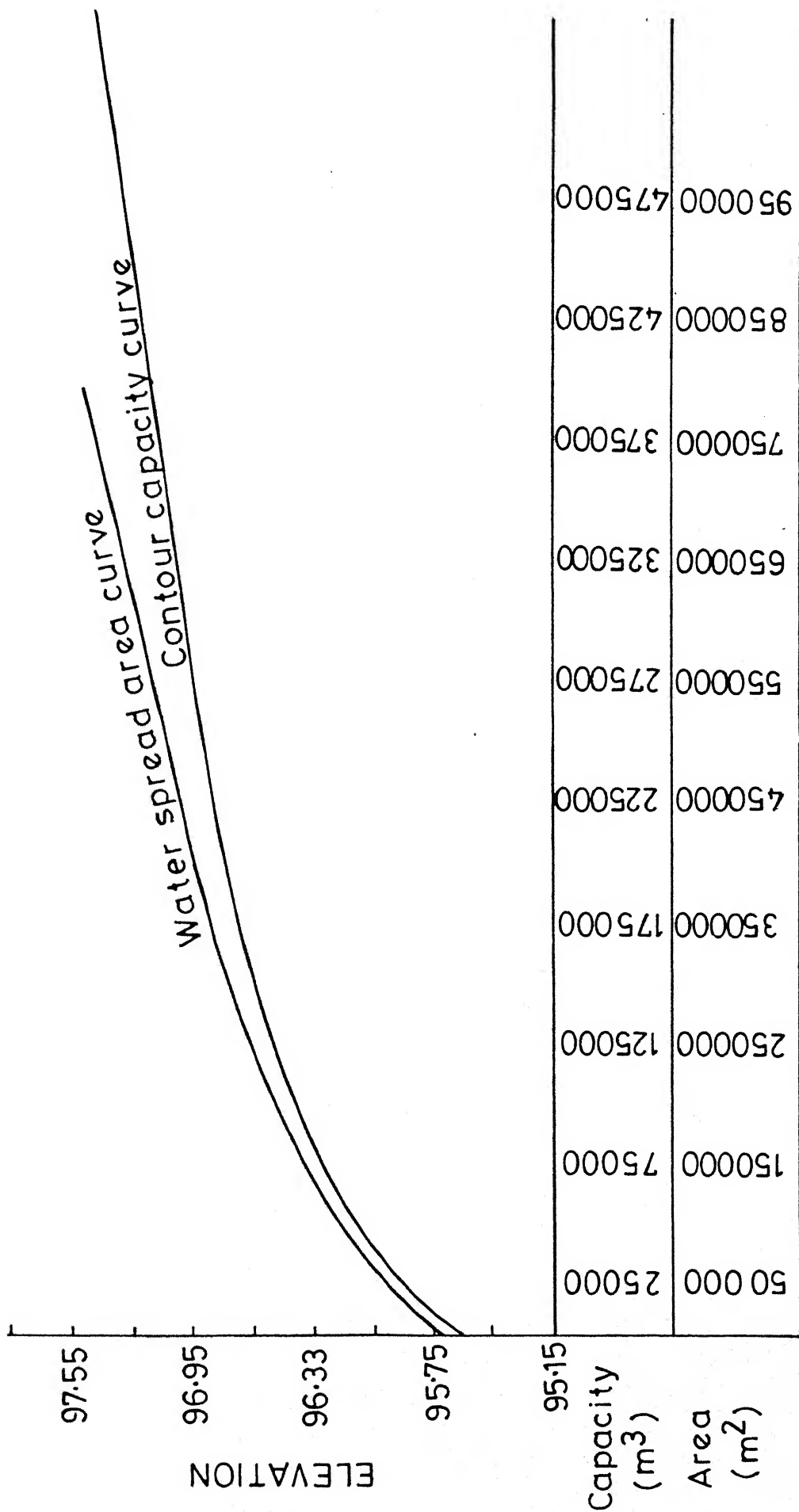


FIG.4.5 URVAKONDA TANK PARTICULARS

(Stage Vs area and Stage Vs Cumulative Volume)

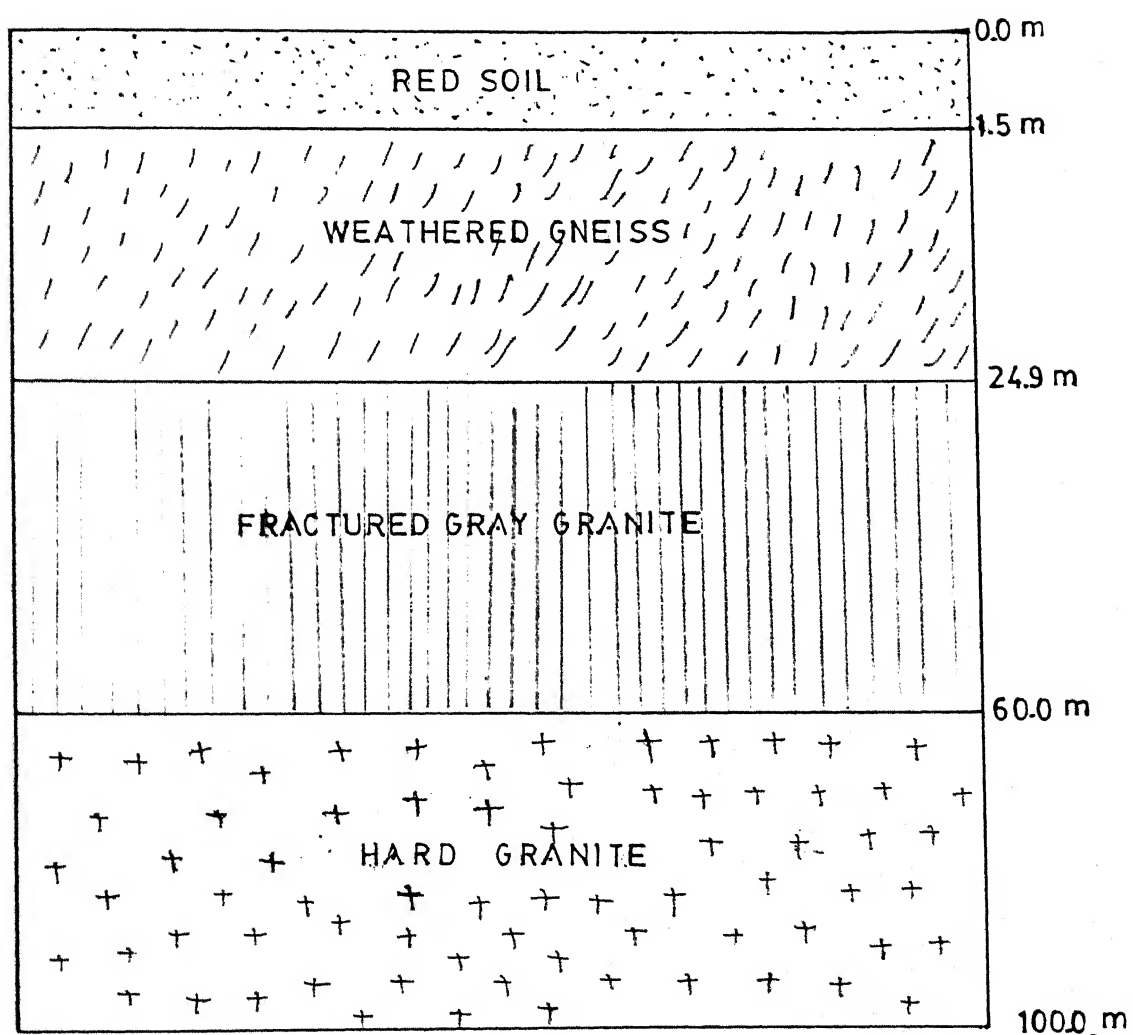


Fig. 4.7 Lithology of Havaligi Basin

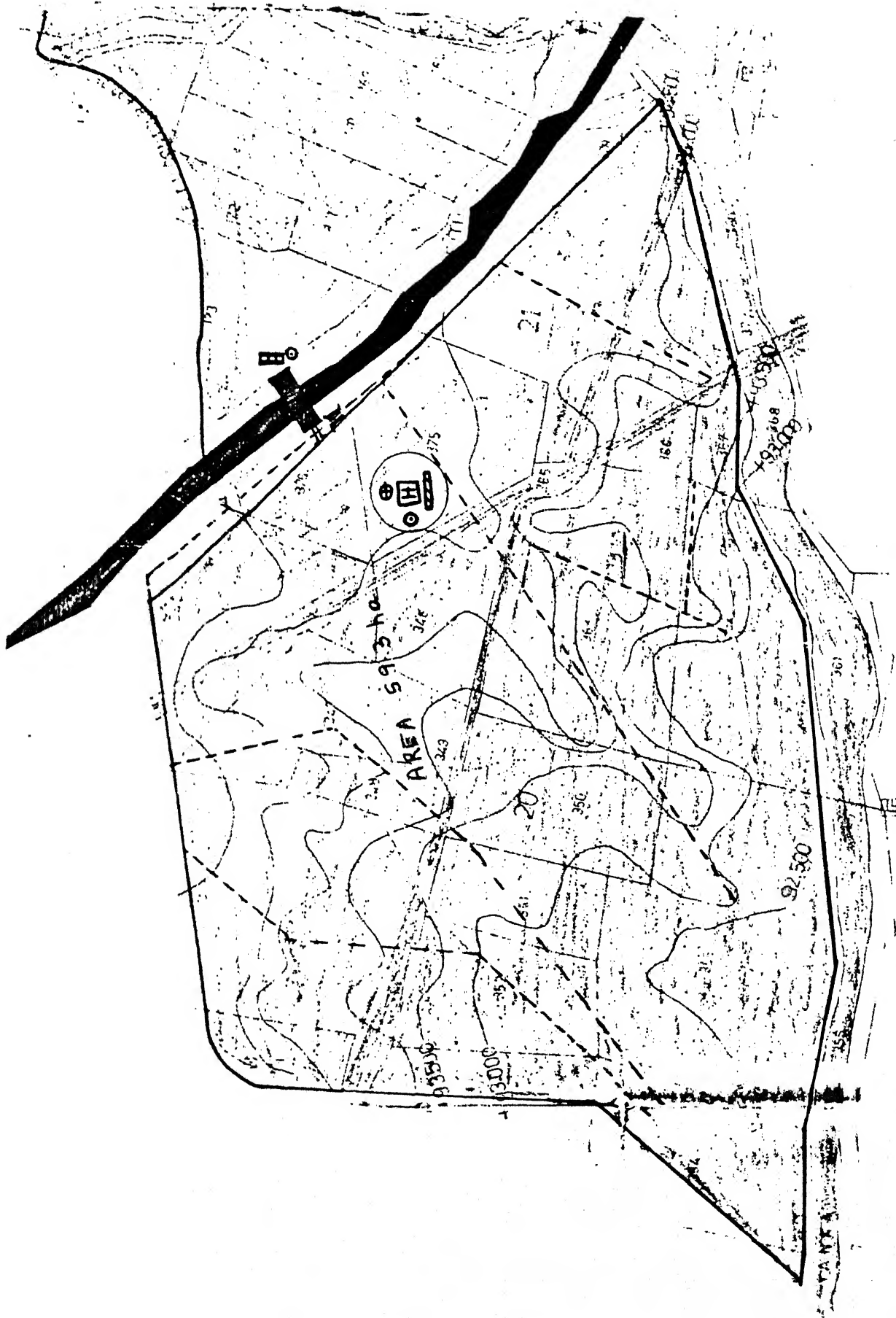


Fig. 4.8 Command Area of Urvakonda Tank



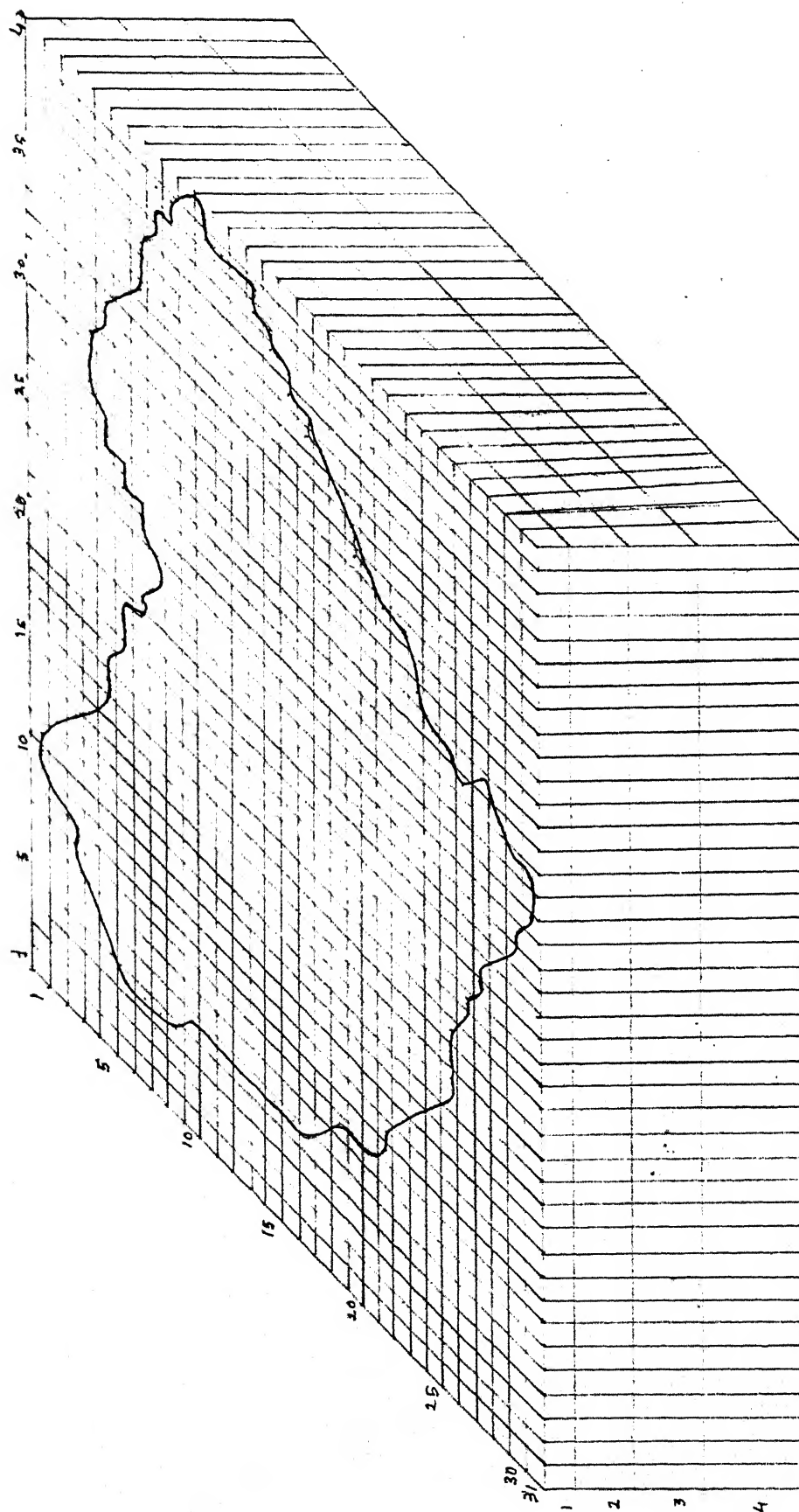
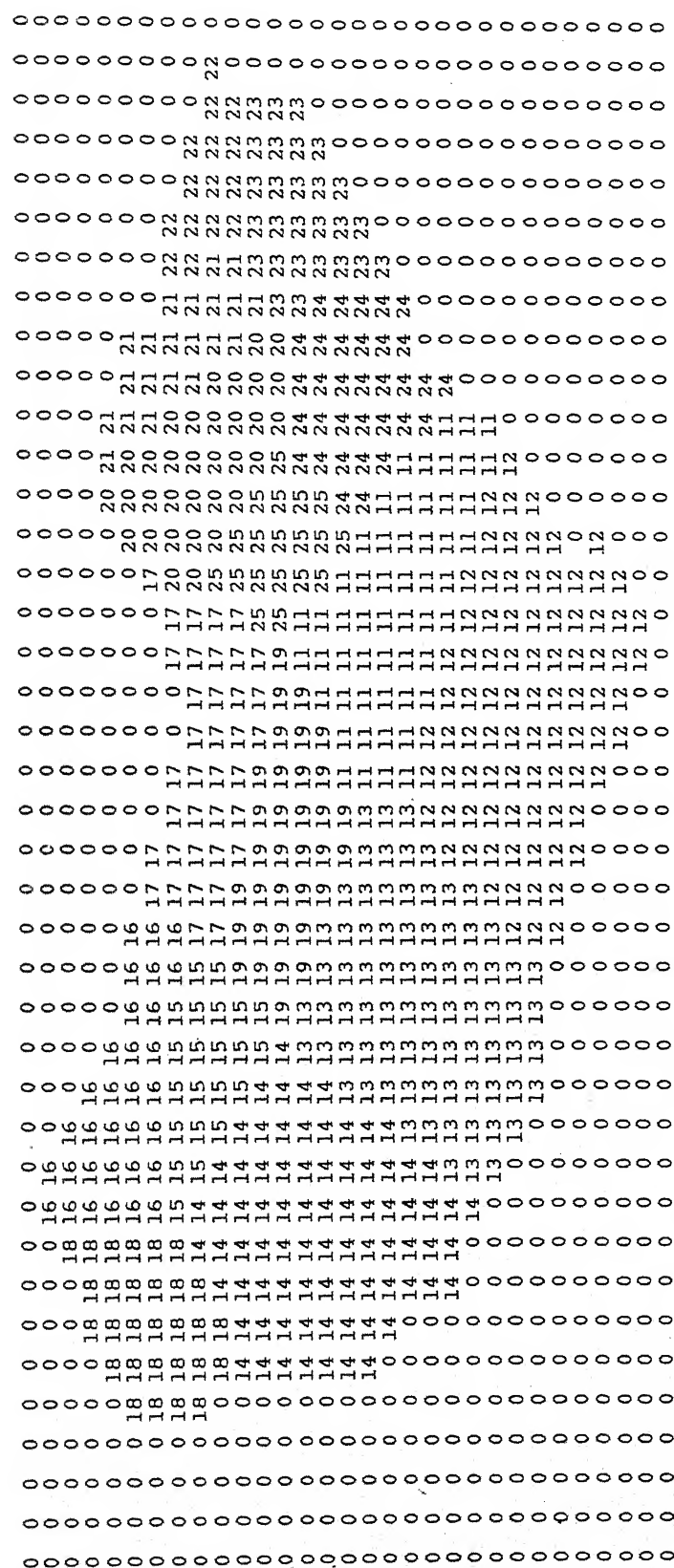


Fig. 4.10 Discretization of Havaligi Basin





**Fig. 4.11 Discretization of Boundary conditions.**

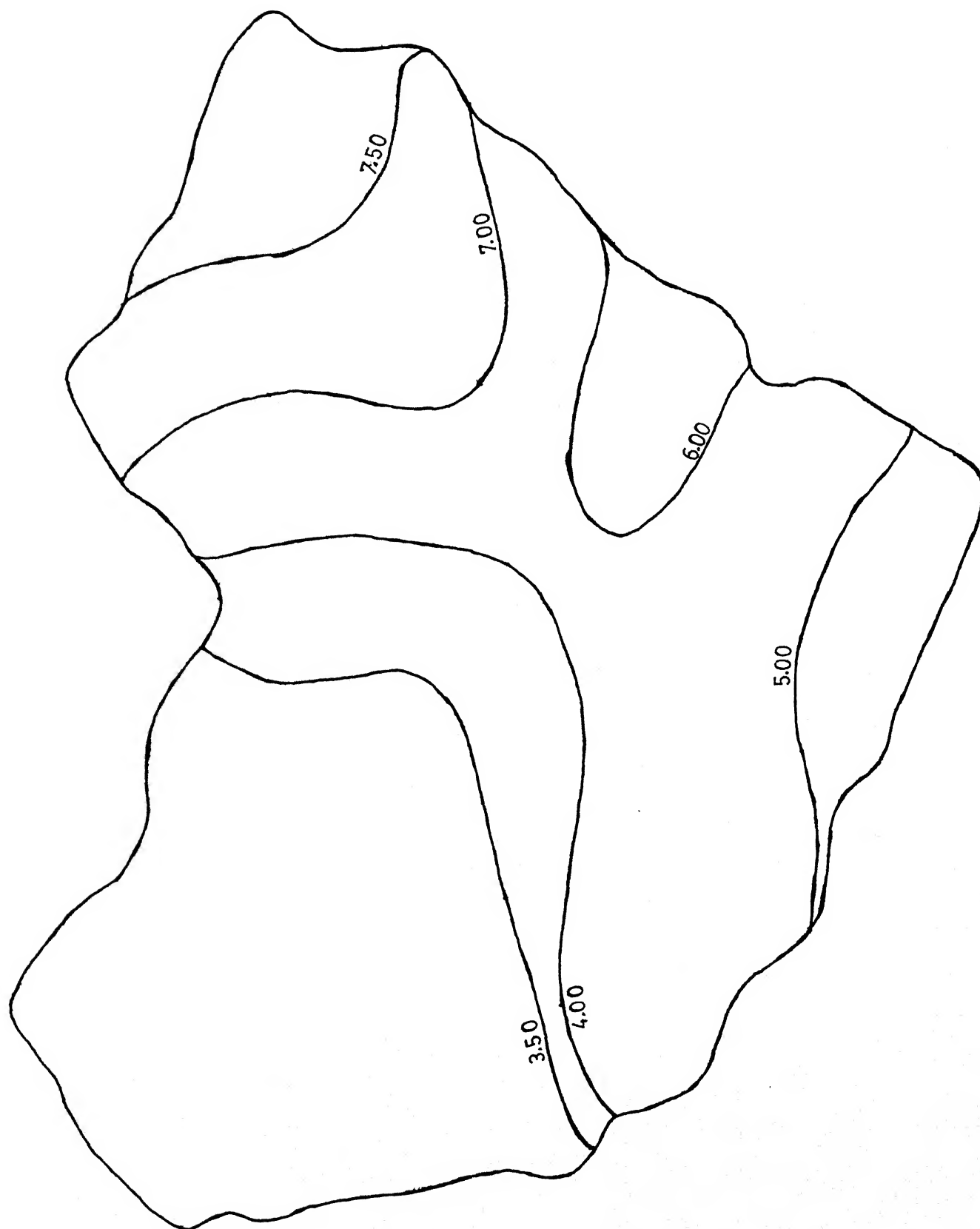


Fig. 4.12 Depth to Water Table Levels in January 1970

## CHAPTER 5

### SUMMARY OF RESULTS, CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

#### 5.1 General:

The need for evaluation of groundwater balance is discussed earlier. Two methods for evaluation viz., the simple hydrologic budget method and the three dimensional finite difference method were presented in detail in the previous Chapter. In this Chapter the results obtained by the models are presented. The summary of results with conclusions and suggestions for further work are included in this Chapter.

#### 5.2 Results of the Hydrologic Budget Method of Analysis:

The general equation for hydrologic budget method has already been explained in the previous Chapter. The two inputs to the basin are rainfall and seepage from canals. The outputs are consumptive use, evaporation from tanks and barren lands, outflows from the tanks. The balance between the input and the output gives the net change in storage.

Using the available data for the years 1979 and 1980 for Havaligi basin, the above quantities are evaluated. Table 5.1 shows the results. In this table, the rainfall volumes to the basin for the corresponding month is calculated using the average rainfall over the basin. The values of consumptive use are obtained by taking into account the various crops grown in the

basin. This includes evaporation from tanks, canals and from barren lands. The outflows from the tanks are available as input data. The quantity of seepage from canals is evaluated by using Darcy's Law in which a bed thickness of 30 cm, a coefficient of permeability of 0.4 m/month, and variable head depending upon the canal water level in that particular month are used. The algebraic sum of rainfall, consumptive use, outflows from tanks and seepage from canals is the net hydrologic budget.

It is seen from the table that a total quantity of 50.75 million cubic metres are added to the basin in the year 1979. Similarly, an amount equal to 32.68 million cubic metres are added. It may also be seen that in some months the net storage comes out to be negative. This means in those months this amount of water is removed from groundwater storage.

### 5.3 Rainfall Recharge to the groundwater:

One of the important component of groundwater budget analysis is rainfall recharge. The method of analysis to get the total recharge to the groundwater table has already been explained in Chapter 4. This total recharge includes rainfall recharge, canal recharge and recharge from other sources. In the present case, recharge only from two sources are considered namely rainfall recharge and recharge from canals. Table 4.8 gives the results of monthly rainfall recharge for the years 1979 and 1980, for each of the polygons into which the basin has been divided. The monthly values should be used rather cautiously as changes in soil moisture were not considered in computing these recharge values. In situations where soil moisture data are not available, it is

desirable to make an annual balance for groundwater recharge rather than monthly balance on the assumption that the soil moisture at the beginning of each year will be the same. In our country, it is perhaps reasonable to assume that the soil moisture in the month of May (just before the onset of monsoon) is the same in each year. By making such an annual balance for groundwater recharge, it is found that the rainfall recharge for the year 1978-79 is 46.27 million cubic metres and for the year 1979-80 it is 40.82 million cubic metres respectively. Similarly the total hydrologic balance is also recalculated for the water years 1978-79 and 1979-80 as 55.56 and 48.28 million cubic metres respectively. The difference between these numbers in each water year is because of seepage from canals which has already been discussed above.

#### 5.4 Results from Three Dimensional Analysis:

The discretization of the basin has been explained in Chapter 4. A total of 31 rows, 40 columns and 4 layers are used. Actually, last layer is a no-flow layer as required by the program. The aquifer characteristics for each of the cells is obtained by using the results of pump test conducted at a few selected points in the vicinity of the basin.

The initial values of the heads used in the analysis are obtained from Fig.4.12. The boundary conditions for the basin are shown in Fig.4.11.

Using the above data, the three dimensional finite difference model was applied to the basin. The various types of results that we get from this model are: (i) the head distribution in different layers at different time periods (ii) the spatial distribution of drawdown at different time periods (iii) the overall volumetric Budget for each stress period. Since, the output is voluminous, it is not possible to give all the results. Table 5.2 gives a sample result of head distribution in layer 3 at the end of December 1979. It is seen from the table that at some points the wells go dry; this is indicated in the table by a code number 1.00000E+30. The digits 9999. in the table represent inactive cells. Most of the tabular values are negative, as the datum was taken at the ground level. The cells in which the heads are more negative numerically are the ones which need attention in planning further pumping patterns.

### 5.5 Conclusions:

The objective of the present study was to implement the Modular Three Dimensional Finite Difference Groundwater Flow model on the mini computer namely Micro VAX II and on the DEC 1090 systems. The second objective was to apply this model to a basin in Anantapur district of Andhra Pradesh. Implementation of the model has been done and the modifications that are required have been indicated. It is to be noted that, for successful application of the model to any basin, voluminous data are required. For example, the aquifer parameters are required not only in the lateral direction but also for different layers. Such

detailed data may not be available for many basins. In this sense, there is a limitation of the applicability of the model. However, where such data are available, the model can be used for reliable results. It is further emphasised that the time varying data such as rainfall, runoff, evaporation, canal flows etc., are available for about ten years for reliable results.

In the present case data for Havaligi basin was available for only two years. Hence the results of application of the model to the basin will only be approximate.

In addition to the detailed three dimensional analysis, hydrologic budget method was also applied to the basin. This method will not take into account the variation in the hydrologic parameters of the basin and thus gives only the overall balance.

#### 5.6 Suggestions for Further Work:

While applying the model, certain anomalies were noticed. For example, at some nodes, it was found that the head values were becoming positive, which indicates a spring. But, on the field no such springs were noticed. This may be because of paucity of the data. It is, therefore, suggested that in future work, in this basin, data over longer periods of time, say ten years, should be used. A part of the time series of the data may be used for the calibration and the remaining for validation.

TABLE 5.1 SUMMARY OF INFLOWS AND OUTFLOWS

YEAR 1979

Ref. No.	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	2.8920	9.3057	7.6232	17.1898	41.0138	7.7745	12.6079	0.0000
2	1.3990	1.4252	0.0000	0.0000	2.0093	7.8050	8.9288	7.7941	9.6452	7.9757	5.0635	1.5521
3	0.0420	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0693	0.0216	0.1429	0.1464	0.1220
4	0.9695	0.0000	0.0000	0.0000	0.0000	0.0000	0.4491	0.6572	1.1089	1.0826	1.1089	1.1089
5	-0.4715	-1.4264	0.0000	0.0000	0.8827	1.5007	-0.8565	9.9836	32.4559	0.7386	8.5068	-0.5652
TOTAL HYDROLOGIC BUDGET OF THE BASIN FOR THE YEAR 1979 = 50.7488 Million Cubic Metres												

YEAR 1980

Ref. No.	JANUARY	FEBRUARY	MARCH	APRIL	MAY	JUNE	JULY	AUGUST	SEPTEMBER	OCTOBER	NOVEMBER	DECEMBER
1	0.0000	0.0000	0.0000	0.0000	4.2855	11.0075	3.8717	6.8160	45.1620	7.5094	0.0000	0.0000
2	1.4977	0.9617	0.7928	0.3956	6.0170	6.5194	8.8945	6.7297	13.4009	6.7219	0.8415	0.7600
3	0.0415	0.0036	0.0000	0.0000	0.0000	0.0000	0.0000	0.0736	0.0181	0.0000	0.0000	0.0000
4	0.9695	0.9695	0.0000	0.0000	0.0000	0.0000	0.7358	0.9826	1.0439	0.9695	0.9565	1.0686
5	-0.5696	0.0042	-0.7928	-0.3956	-1.7315	4.4881	-4.2870	0.9953	32.7869	1.7571	0.1151	0.3086
TOTAL HYDROLOGIC BUDGET OF THE BASIN FOR THE YEAR 1980 = 32.6788 Million Cubic Metres												

Ref. No. 1. Total Rainfall Volumes (Million Cubic Metres)  
 2. Total Evaporation and Evapotranspiration (Million Cubic Metres)  
 3. Tank Outflows (Million Cubic Metres)  
 4. Total Quantity of Canal Seepage (Million Cubic Metres)  
 5. Net Hydrologic Balance (Million Cubic Metres)



TABLE 5.2

HEAD IN LINES 3 AT END OF REVERSE 1000

1	2	3	4	5	6	7	8	9	10
0 1	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 2	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	1.0000E+30
0 3	9999.	9999.	9999.	9999.	9999.	9999.	9999.	1.0000E+30	-30.79
0 4	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-32.59	-35.21
0 5	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-34.55	-38.17
0 6	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-30.64	-34.49
0 7	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-34.64	-33.33
0 8	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-25.83	-37.79
0 9	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-38.25	-18.59
0 10	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-35.48	-33.75
0 11	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-24.93	-21.72
0 12	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-25.00	-33.86
0 13	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-21.08	-24.39
0 14	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-24.92	-32.77
0 15	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-17.16	-23.35
0 16	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-21.99	-36.66
0 17	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-15.95	-21.27
0 18	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-25.11	-38.42
0 19	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-14.27	-24.34
0 20	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-27.33	-33.42
0 21	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-8.975	-25.00
0 22	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-33.02
0 23	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 24	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 25	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 26	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 27	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 28	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 29	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 30	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 31	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.

## HEAD IN LAYER 3 AT END OF DECEMBER 1979

	11	12	13	14	15	16	17	18	19	20
0 1	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 2	1.0000E+30	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 3	-35.19	-41.93	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 4	-40.17	-33.89	-43.62	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 5	-33.22	-40.97	-36.61	-41.92	9999.	9999.	9999.	9999.	9999.	9999.
0 6	-40.07	-35.76	-40.29	-34.64	-40.93	-34.06	1.0000E+30	9999.	9999.	9999.
0 7	-31.41	-33.09	-33.97	-38.74	-37.99	-35.70	-38.37	1.0000E+30	-36.34	9999.
0 8	-39.58	-37.55	-34.71	-33.12	-34.07	-28.07	-32.85	1.0000E+30	-30.41	1.0000E+30
0 9	-25.05	-24.10	-26.85	-32.10	-34.17	-28.71	-31.35	-29.00	-30.67	1.0000E+30
0 10	-34.81	-39.04	-32.29	-29.16	-33.44	-28.45	-30.00	-27.84	-29.36	-18.19
0 11	-26.62	-29.68	-25.57	-31.76	-33.83	-24.75	-33.35	-27.36	-30.45	-26.33
0 12	-28.94	-38.46	-30.43	-29.03	-29.34	-30.75	-30.38	-29.55	-29.69	-22.10
0 13	-29.23	-27.47	-24.08	-34.62	-39.43	-26.98	-30.59	-27.80	-36.37	-27.85
0 14	-26.25	-41.68	-29.08	-27.66	-29.63	-35.37	-31.36	-30.63	-31.67	-24.17
0 15	-28.43	-25.67	-26.75	-37.44	-41.61	-25.60	-33.57	-30.03	-37.58	-27.80
0 16	-29.58	-40.57	-24.79	-28.22	-30.04	-39.90	-30.74	-32.17	-33.13	-27.97
0 17	-25.51	-27.31	-34.78	-34.87	-39.62	-25.41	-34.24	-31.29	-35.16	-29.78
0 18	-32.77	-35.74	-24.05	-32.91	-32.12	-41.08	-29.09	-32.82	-33.44	-29.74
0 19	-24.30	-32.00	-34.00	-31.42	-39.70	-28.00	-35.21	-30.22	-32.44	-28.11
0 20	-32.93	-29.16	-26.52	-27.36	-27.59	-36.90	-26.78	-29.97	-35.22	-28.64
0 21	-20.96	-31.74	-30.83	-31.22	-40.43	-28.96	-29.53	-29.36	-34.50	-24.91
0 22	-27.72	-26.06	-25.96	-27.35	-30.83	-32.27	-29.22	-31.34	-31.87	-28.42
0 23	-21.40	-33.11	-30.60	-27.70	-42.95	-28.19	-31.86	-27.70	-38.90	-24.01
0 24	9999.	1.0000E+30	-28.14	-29.62	-28.35	-33.40	-28.66	-35.58	-34.78	-33.07
0 25	9999.	9999.	-28.49	-28.67	-39.45	-34.13	-30.99	-26.55	-34.97	-27.54
0 26	9999.	9999.	9999.	9999.	9999.	9999.	1.0000E+30	1.0000E+30	-34.94	-35.12
0 27	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	-35.53	-24.90
0 28	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 29	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 30	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.
0 31	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.	9999.

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